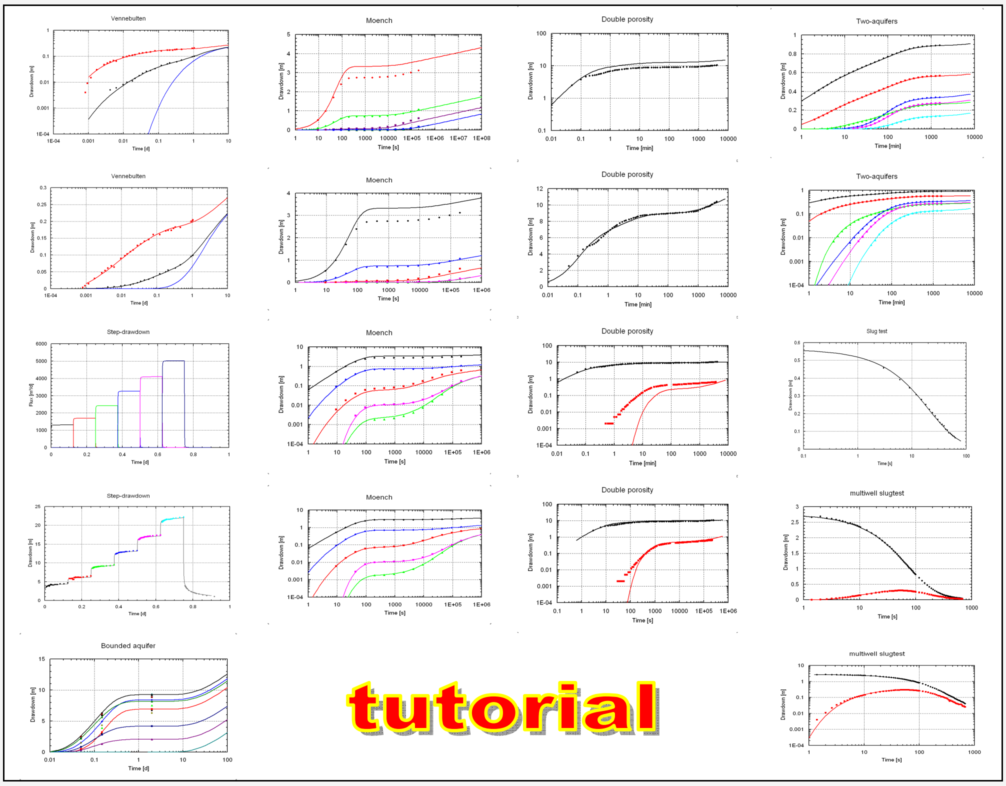
# Modeling with MLU

applying the multilayer approach to
aquifer test analysis





Kick Hemker & Jeff Randall

Amsterdam - Seattle

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# 1. Introduction

Inverse modeling, model calibration and aquifer test analysis are all very much the same when drawdown data are used to find the best values for the hydraulic properties of the aquifer system around a pumping well. When several layers are involved, as in multi-aquifer systems, data analysis is more than just parameter estimation, but also requires conceptual modeling and parameterization. Conceptual modeling (model finding) is searching for the best or simplest conceptual hydrogeologic model that is consistent with the measured drawdowns. Parameterization is identifying the best set of parameters that can be estimated with field data.

Conceptual modeling and parameterization are essentially fixed in classical aquifer test analysis based on the assumptions of the applicable analytical solution. However, MLU for Windows (MLU: Multi Layer Unsteady state) offers more flexibility because the aquifer system can be conceptualized as a layered model. An MLU model can consist of up to 80 aquifer layers with horizontal flow separated by aquitards or resistance layers controlling vertical flow. All layers have their own thicknesses and hydraulic properties and together with the boundary conditions form the MLU aquifer system.

In addition to parameter estimation, MLU can be used to predict drawdown given a conceptual hydrogeological model with a given set of hydraulic properties. This can be very useful in designing wells, well fields and aquifer tests.

Although all of this flexibility is useful it may not be immediately clear how to convert a real world aquifer test into an MLU multi-layer model. The primary objective of this tutorial is to help users develop the best MLU models by presenting information on how MLU handles groundwater flow, practical advice on how to conceptualize the aquifer system, and specific examples of aquifer test analysis using MLU.

This tutorial is intended as a companion document to the MLU User's Guide (*Well flow modeling in multilayer aquifer systems*, Hemker & Post), which contains a complete description of the MLU software including the details of data entry and running the model. After working through this tutorial, we recommend gaining additional experience by building a few aquifer test models with known results selected from textbooks, publications or other analytical or numerical models.

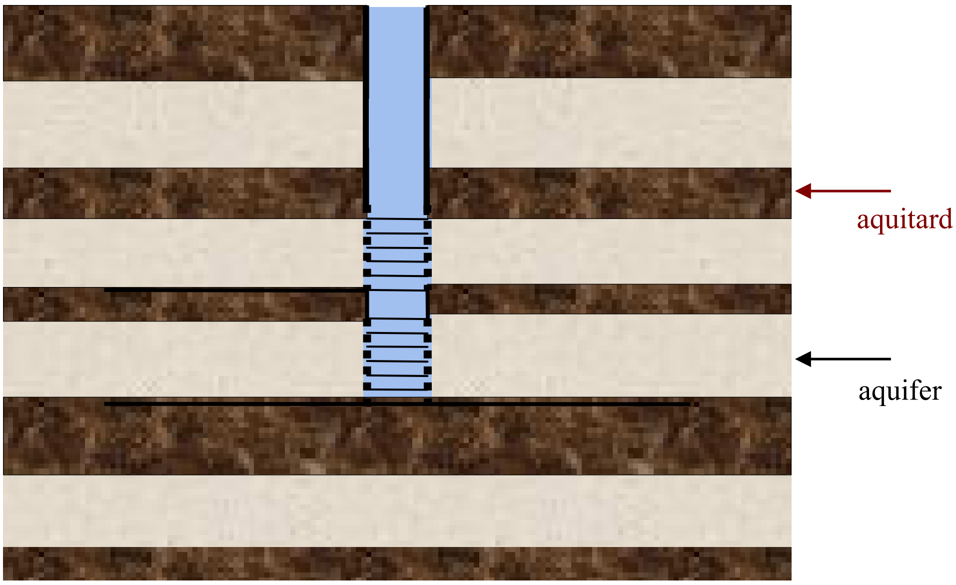
Many of the specific examples presented in Section 5, MLU Aquifer Test Examples, are taken from the publication *Analysis and evaluation of pumping test data* by Kruseman & De Ridder (reprinted in 2000 and available online – see References section). This publication is often referenced because it is an excellent compendium of traditional aquifer test analysis methods.

## 2. MLU Multi-Layer Modeling Approach

MLU can be used to build fully 3-dimensional models of transient well flow. It is based on an analytical solution for transient well flow in layered aquifer systems, as described in Hemker (1999). The analytical solution computes hydraulic head drawdowns, but the vertical flow component and time also have some numerical aspects.

The "layer concept" in hydrogeology is based on the fact that hydrostratigraphic units typically have a large horizontal extent and a relatively small thickness. In MLU a layer is assumed to be homogeneous, have a uniform thickness and an infinite horizontal extent. A layered aquifer system is a stack of layers that all respond to the pumping of a well. The system is bounded at the top and at the base.

3D groundwater flow in MLU can be modeled as horizontal flow in a number of aquifer layers with vertical flow through aquitards between these layers. When the hydraulic conductivity contrast of the layers is large (say 1:100 or more) flow is mainly horizontal in the high-conductivity layers (aquifers) and mainly vertical in the low-conductivity layers (aquitards). A system that consists of a series of alternating aquifers and aquitards is referred to as a multi-aquifer system.



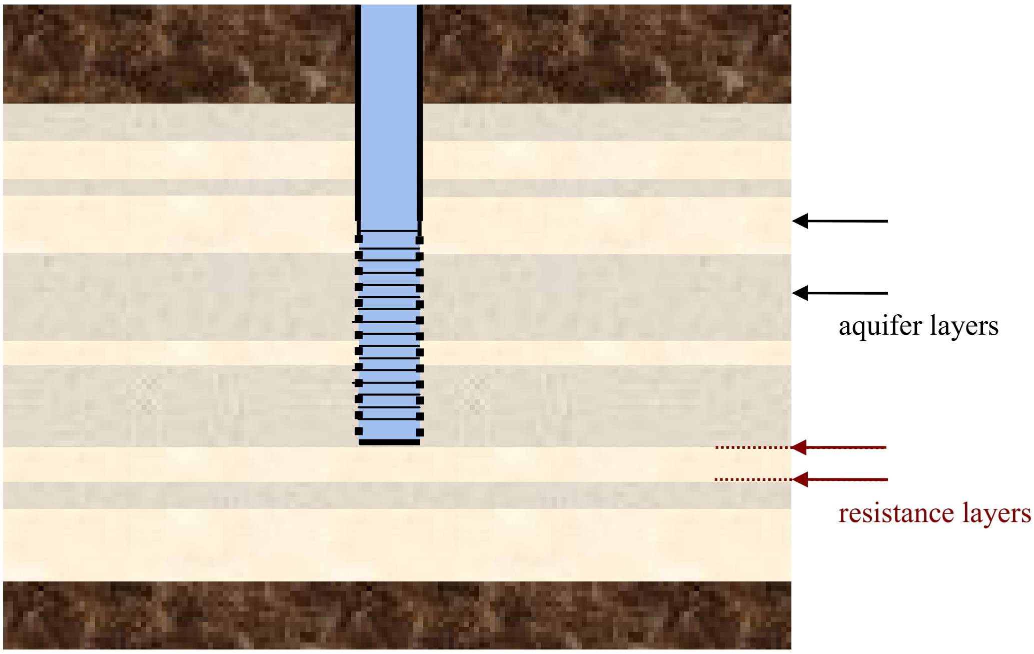

**A multi-screened well in a multi-aquifer system. All aquifers are separated by aquitards.**

In some cases it is important that the vertical flow component within an aquifer layer is modeled explicitly. Examples include a partially penetrating pumping well in a homogeneous aquifer or an aquifer with a number of horizontally extensive subunits with differing hydraulic properties. In such cases the aquifer can be modeled as a number of aquifer sublayers separated by zero-thickness resistance layers and is referred to as a layered or stratified aquifer.

Similarly, any aquitard can be modeled as a layered aquitard, consisting of a number of aquitard sublayers separated by thin, low transmissivity aquifer layers. Splitting an aquitard into sublayers can be useful:

- to allow some horizontal flow within an aquitard
- to account for layers with different hydraulic properties within an aquitard
- to compute head values within an aquitard.

The conductivity contrasts within layered aquifers and layered aquitards are typically less than within multi-aquifer systems.




**A partially penetrating well in a stratified confined aquifer. Layers with higher and lower conductivities are separated by zero-thickness resistance layers.**

In a horizontal plane, MLU calculates drawdown as a continuous function of the distances to the pumping wells. For each pumping well and each observation well the position (x- and y-coordinates) can be accurately specified.

In the vertical dimension, MLU only calculates drawdown for a limited number of depths. The solution technique is based on the multi-layer approach. The multi-layer approach means that the computed drawdowns don't vary continuously in the vertical direction, and only a single drawdown is computed for each layer with horizontal flow representing the drawdown in the middle of the layer. The discretization of the z-coordinate to represent the aquifer system as a number of layers is essentially similar to the vertical discretization of finite difference models.

Drawdown is an analytical function of time, so it can be computed for any single moment, without bothering with time steps as in numerical models. However, there is a numerical procedure involved in the computations with regard to time: the Stehfest algorithm (Stehfest, 1970). A short description of this technique is given by Hemker & Maas (1987).

In addition to 3D groundwater flow, MLU can also be used to model Q3D (quasi-3D), 2D, Q2D and 1D (radial) groundwater flow. In a Q3D and a Q2D model only horizontal flow in the aquifers and vertical flow in the aquitards is considered.

## Computing drawdowns

For all computations MLU uses the multi-layer uniform well-face drawdown solution (Hemker, 1999). It is based on horizontal flow within aquifer layers and vertical flow between aquifer layers. Drawdown is assumed invariant with depth within any aquifer layer. As a rule vertical flow occurs within aquifer layers, but it is disregarded in the MLU solution technique. This approximation is the same as assuming that the vertical hydraulic conductivity of each aquifer layer is infinitely large. Drawdowns are only computed within aquifer layers, not within aquitards.

Vertical flow within aquitard layers is computed analytically as one-dimensional flow. Drawdown varies continuously in the vertical dimension, but values are not presented by MLU. When the drawdown at some level within an aquitard is required, the aquitard should be split into two or more layers separated by thin, low-transmissivity aquifer layers that simulate horizontal flow within the aquitards. Drawdowns are computed for these layers within an aquitard.

The multi-layer solution technique also implies that the layer thicknesses and conductivities themselves are not involved in the drawdown computations. It is only their combinations as transmissivities and as vertical hydraulic resistances that play a role in the computations of horizontal flow and vertical flow, respectively. Base elevations, thicknesses and conductivities are not essential conceptual model inputs, but are only part of the aquifer system inputs to assist the user in building the MLU model. This also explains why transmissivities, vertical resistances and aquifer and aquitard storativities are the only hydraulic properties that can be determined by MLU's automated parameter estimation method.

## Well-face fluxes

In analytical models of 3D well flow it is generally assumed that the flux through the screen is uniformly distributed. This means that each unit length of screen passes an equal part of the total discharge rate. In the case of a stratified aquifer the uniform well-face flux condition is replaced by a uniform well-face gradient where the well-face flux is proportional to the layer conductivity. The weakness of this internal boundary condition is that vertical flow in the aquifer will cause a drawdown that varies along the screen. So, an alternative condition is to assume that the drawdown at the well screen is the same at any depth at any time. This condition is called the uniform well-face drawdown condition.

In MLU a single pumping well can be screened in more than one aquifer layer. The distribution of the user-specified discharge rate over the screened layers is governed by the uniform well-face drawdown condition. This distribution varies during a pumping test and depends on the aquifer system characteristics and also on the well screen properties (well screen and casing radii and skin). The solution method requires that a non-zero well screen radius is known. Changes in wellbore storage are another cause for the aquifer layer discharge rates to vary in time. The resulting variation of the discharge rate per layer can be displayed in a well-face flux graph, which is one of the two possible time graph curve types that MLU can generate.

MLU computes the aquifer discharge through each screen for each well separately. This implies that the discharge rate in any screened layer of any pumping well is not affected by the drawdown caused by any other pumping well.

## Assumptions

Important assumptions of MLU are:

- The aquifer system consists of a stack of aquifer layers with horizontal flow separated by aquitards or resistance layers that control vertical flow
- All layers are of infinite horizontal extent
- All aquifer layers are homogeneous and isotropic with respect to transmissivity
- All aquifer layers are homogeneous with respect to storativity
- All aquitard layers are homogeneous with respect to vertical resistance and storativity
- Only saturated groundwater flow is considered
- The top and base conditions are either no-drawdown or no-flow boundaries
- Well screens fully penetrate one or more aquifer layers
- Observation wells are screened in only one aquifer layer
- Only drawdown (or build-up) as a result of pumping is considered
- A well is pumped at a constant or step-wise changing rate
- In case of a multi-layer screened well the discharge rates of individual layers are based on a uniform drawdown at the well screen.

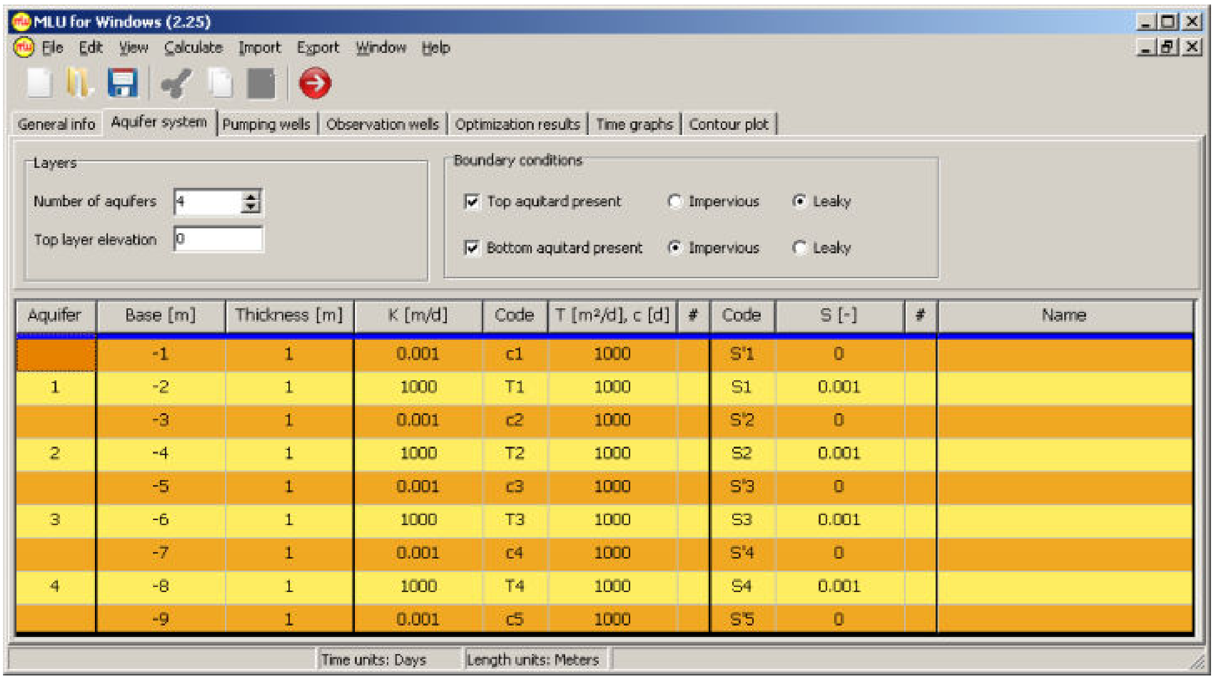
Other (implicit) assumptions are:

- Darcy's law is valid, except for turbulent flow near the well screen
- Turbulent flow near the well screen can be modeled by a thin skin
- The water removed from storage is discharged instantaneously with decline of head
- The influence of unsaturated zone flow on drawdown is negligible
- A rising or falling water table doesn't affect the transmissivity or storativity of a water-table aquifer, nor the vertical resistance or storativity of a water-table aquitard
- The existence of a seepage face at a water-table well is ignored
- Effects of horizontal-deformation processes (Noordbergum effect) are negligible
- No mutual effects of pumping wells. The discharge rate in any screened layer of any pumping well is not affected by the drawdown caused by any other pumping well.

# 3. Basic Guidelines for Modeling with MLU

It should be clear by now that regardless of how an aquifer system is represented in MLU, it is always a stack of horizontal flow aquifer layers separated by aquitard or resistance layers that regulate vertical flow. In the "Aquifer system" tab (part of the MLU user interface) the layering is illustrated by alternating yellow and orange rows. The layers with horizontal flow (yellow rows) are numbered from top to base.

When a new aquifer system is set up (File | New), the required number of aquifers can be selected both in the "General info" tab and in the "Aquifer system" tab. In the latter case one has to click in the table to effect the new setting. This "number of aquifers" should be read as "the number of aquifer layers with horizontal flow". The Aquifer system table is found in the "Aquifer system" tab. In a new aquifer system table the cells will have default values, like 1, 1000 or 0.001, and the names of the layers and the #-columns are blank.




| Aquifer | Base [m] | Thickness [m] | K [m/d] | Code | T [m²/d], c [d] | # | Code | S [-] | # | Name |
|---|---|---|---|---|---|---|---|---|---|---|
| | -1 | 1 | 0.001 | c1 | 1000 | | S'1 | 0 | | |
| 1 | -2 | 1 | 1000 | T1 | 1000 | | S1 | 0.001 | | |
| | -3 | 1 | 0.001 | c2 | 1000 | | S'2 | 0 | | |
| 2 | -4 | 1 | 1000 | T2 | 1000 | | S2 | 0.001 | | |
| | -5 | 1 | 0.001 | c3 | 1000 | | S'3 | 0 | | |
| 3 | -6 | 1 | 1000 | T3 | 1000 | | S3 | 0.001 | | |
| | -7 | 1 | 0.001 | c4 | 1000 | | S'4 | 0 | | |
| 4 | -8 | 1 | 1000 | T4 | 1000 | | S4 | 0.001 | | |
| | -9 | 1 | 0.001 | c5 | 1000 | | S'5 | 0 | | |

**The three components of the MLU "Aquifer system" tab: Layers, Boundary conditions and Aquifer system table.**

The first layer at the top of the aquifer system can be either an aquifer or an aquitard. When an aquifer is present, the aquifer system boundary must be impervious. When an aquitard is present, the boundary condition is either an impervious (no-flow) boundary or a no-drawdown boundary. An aquifer on top in combination with a no-drawdown boundary would exhibit no drawdown, therefore it would have no effect on the aquifer system and thus not allowed in MLU. Usually an impervious boundary is assumed at the base of the system, but MLU allows the same conditions here as at the top.

There are some limitations to the values that can be entered in the aquifer system table. Negative values are only allowed for the top layer elevation and for the base of each layer. Conductivities, transmissivities and vertical resistances are always positive. Zero values are used in only two cases:

- The thicknesses of resistance layers within a stratified aquifer are entered as zero
- In most models aquitard storativity is assumed zero, but positive values are allowed.

Apart from using the mouse left-click to select and open a particular cell for input, one can navigate through the aquifer system table using the keyboard arrow keys and Enter key to open and close a cell.

Changing the number of aquifers adds or removes aquifers and aquitards at the base of the system. Right-click a particular row within the table if you want to insert or delete a layer at that particular depth within the system. Choose "Insert" to add an aquifer and overlying aquitard. All properties of the new layers are copied from the selected row. Choose "Delete" to remove the selected aquifer and overlying aquitard.

It is possible to Copy and Paste within the aquifer system table, between the aquifer system tables of different models and between the aquifer system table and other software such as spreadsheets. Blocks of information can be selected by using the cursor keys while keeping the Shift key pressed. The codes in the code columns are copied, but disregarded when pasted into the aquifer system table.

## Pumping wells and observation wells

The required model input for pumping wells and observation wells (see MLU user interface "Pumping wells" and "Observation wells" tabs) is straight forward and explained in the Help file and in the User's Guide. A pumping well screen is assumed to fully penetrate one or more aquifer layers. Therefore, a partially penetrating well screen can be a good reason to split an aquifer into several sublayers in such a way that the screen is located in one or more layers. Each pumping well in a multi-well system can have its own influence on how to divide the aquifers into model layers.

An observation well can only be screened in a single aquifer layer. When vertical flow components are relatively large, e.g., near a partially penetrating well or in a water-table aquifer, it may be useful to divide the aquifer in such a way that the screened interval of the observation well corresponds to a layer (or the end of a piezometer to the center of a layer).

In MLU time always starts at t = 0, and at t = 0 drawdown in all layers is zero (initial condition). The smallest time that can be entered in MLU is $1 \times 10^{-6}$ days ($1.44 \times 10^{-3}$ minutes; $8.64 \times 10^{-2}$ seconds).

## Storage and leakage

When a leaky (semi-confined) aquifer is pumped, the pumped water comes from different sources that gradually change with time. In the case of a single aquifer with an overlying aquitard and an impervious base, five different sources can be identified. In the order of their effects becoming noticeable, these sources are:

"a" – Wellbore storage (due to the falling water level in the well casing)
"b" – Aquifer storage (due to the compressibility of the aquifer material and that of water)

“c” – Aquitard storage (due to the compressibility of the aquitard material and that of water)
“d” – Leakage through the aquitard (from overlying surface water or from an adjacent aquifer)
“e” – Water released by a falling water table in the aquitard.

Many analytical solutions account for only a few of these sources. The simplest of them is the steady-state solution where all water comes from leakage. Such solutions require that the unaccounted for sources are negligible, which limits their applicability to a restricted time period.

In analytical solutions, aquitard storage is usually assumed negligible and consequently its effects are often neglected or attributed to the storativity of adjacent aquifer layers. Aquitard storage is used in two of the Section 5 examples (Dalem, Schroth).

MLU uses a generalized analytical solution, which as a rule accounts for the sources “a” through “d”. The water released by a falling water table can also be simulated by MLU, but such conditions require an additional layer at the top of the system. This ‘water-table layer’ or ‘phreatic layer’ is characterized by a low transmissivity and a high storativity (the specific yield). Further details about ‘water-table layers’ are discussed in the Section 5 examples (Moench, Two-aquifers, Vennebulten).

To simulate flow conditions for relative simple cases, each of the sources can be ignored in MLU:

Ignore “a” – set the casing radius of the pumping well rc = 0 to remove effects of wellbore storage
Ignore “b” – set the aquifer storativity S = 0 to remove effects of aquifer storage
Ignore “c” – set the aquitard storativity S’ = 0 to remove effects of aquitard storage
Ignore “d” – set the top boundary conditions to impervious to remove effects of leakage.
Ignore “e” – no water-table layer required.

Therefore, a major advantage of the MLU approach is that the limiting conditions with respect to a valid time period do not exist. Any period from the very start until the end of the pumping can be used to analyze the observed drawdowns.

## Measured drawdowns

When field measurements are used in aquifer test analysis, the observed levels may have to be corrected for external influences such as tides, barometric changes, rising or falling regional water levels, etc. A spreadsheet can be very useful for this purpose (e.g., Halford, 2006).

The number of observations that can be used in MLU is limited to 100 observations wells with a maximum of 1000 measurements for each well. When pressure transducers and data loggers are used to monitor the aquifer test, data reduction will be required. Usually only 4 to 10 drawdowns measurements per log cycle are sufficient to characterize the shape and position of a drawdown curve.

## Time and length units

The time units in MLU can be seconds (s), minutes (min), hours (h), days (d) and years (yr). The required unit can be selected after right-clicking the appropriate text on the Status bar (lowest part of the MLU screen) or changing Time units in the “General info” tab of the main MLU window.

When entering all time related data into an MLU model it is critical to get the time units set correctly and that all entered data be based on the selected unit of time. For example, if the time units are “days” then the time-drawdown data must be entered in days, the pump start time(s) must be in days, the pump discharge must have units of $m^3$/day (or $ft^3$/day), and any parameter (e.g., T) entered into the “Aquifer system” tab table must have units of $m^2$/day (or $ft^2$/day). However, a different time unit can be selected at any point during data entry. So it is possible to enter transmissivities in $m^2$/day, discharge rates in $m^3$/h and observation times in minutes: just select the required time unit before entering the data.

MLU associates the selected time unit with the raw input data at the time of entry. Changing time units after data entry causes MLU to physically convert all time related values based on the new units rather than assigning new time units to the original data. This also allows the user to display the raw data and model results with other time units without re-running the model.

If you enter any time related data (especially the time-drawdown data) with improper time units, you do not have to re-enter them individually: just copy the block of data (shift + arrow keys to select, ctrl + c to copy), adjust the MLU Time units and paste the data back at the same location (select the upper left cell, ctrl + v to paste).

Internally MLU converts all time related data to time units of days for computation and stores the converted data in the *.mlu project file along with the user selected Time units. When the saved *.mlu file is opened, the day based data are loaded into memory and MLU makes unit conversions based on the user selected Time units to display all time related data and labels in the model.

MLU internally calculates parameters based on time units of days. Results summarized in the “Optimization results” tab are also based on days, except when “Time units” is set to seconds. All other MLU displayed time-related data, parameter values, and labels are based on the user selected Time unit.

The length units in MLU can be meters (m) or feet (ft). The required unit can be selected after right-clicking the appropriate text on the Status bar (lowest part of the MLU screen) or setting a default unit on the Menu bar (Calculate | Preferences). Changing length units causes MLU to physically convert the length related values of all input data in a similar way as changing time units.

## Aquifer test analysis overview

As discussed in the Introduction, aquifer test analysis using MLU is more than just parameter estimation, it also involves conceptual modeling and parameterization.

Although “parameter” is in general just the value of a hydraulic property of some layer, when we speak about aquifer test analysis its meaning is often restricted to the values of the properties that we choose to optimize.

The iterative process of parameter estimation is automated in MLU, but the other parts of the analysis (conceptual modeling and parameterization), which also are often iterative processes, are left to the user. Details about the way that MLU can be used for parameter estimation and the interpretation of the results are presented in the MLU User’s Guide, Section “Results: Parameter optimization”. Here we will only add some practical details.

Before parameters can be estimated, a model has to be set up. Each layer must be assigned a transmissivity and storativity value or a hydraulic resistance and (possibly zero) storativity value even if the parameter is not to be optimized. To help the parameter optimization find the global best solution (rather than a local solution) the user should enter realistic values for all the parameters to be optimized and all other model properties including pumping well and observation well parameters as needed.

Borehole data and experience are helpful when estimating horizontal conductivities and transmissivities. Vertical conductivities of aquifers (needed to find the hydraulic resistance of resistance layers) are often estimated between 5% and 50% of the horizontal conductivity, with the lowest values for clearly layered formations. Resistances of aquitards are much harder to estimate ranging from tens to even hundreds of thousands of days.

To estimate initial values for the storativities of sandy aquifers, the “Van der Gun equation” (Van der Gun, 1979) is often used in the Netherlands:

> $S = 1.8 \times 10^{-6} (d_2 - d_1) + 8.6 \times 10^{-4} (d_2^{\,0.3} - d_1^{\,0.3})$ , where $d_1$ (m) is the depth to the top of the aquifer below ground level and $d_2$ (m) is the depth to the aquifer base below ground level.

Aquifer test analysis is an iterative process. First try the simplest hydrogeologic model. If the MLU computed drawdown doesn't fit the test data, try to understand what causes the differences and then adjust the model. When modification of the hydraulic property values doesn't sufficiently help, vertical discretization adjustments may be required. In MLU it is easy to add a layer and see how it affects the results. Because there are usually many ways to proceed, it depends on the insight of the modeler (and some luck) to find output that fits the data. A “better conceptual model” is an aquifer system with a set of parameters which generates drawdowns that fit the field data better, or can fit the data with fewer layers. Professional judgment and intuition are also often important in finding a good model or, as K&dR (Kruseman & De Ridder, 2000) write on page one: “Analyzing and evaluating pumping test data is as much an art as a science”.

Parameter optimization in MLU usually results in a least-squares solution. When a solution is found, the iterative calculations stop with the remark “parameters found”. Clicking the optimize button again sometimes helps find a more precise solution. Sometimes, the calculation process may stop with the remark: “calculation broken off (stopping criteria too small)”. In this case the optimization results are not presented and a new computation is required, starting with a slightly changed set of initial values. Switching between “log drawdown curve fitting” and “linear drawdown curve fitting” also helps. Frequently switching between these alternative optimization methods can be useful when searching for a good fit.

When a least-squares solution is found, but the model fit is very poor, it is possible that the initial values of the parameters are too far off (resulting in a local optimization). Some manual trial and error parameter estimation can help find better initial values. Another useful approach is to start optimizing only one or two parameters (usually the transmissivity of the pumped aquifer) and then gradually increase the number of active parameters.

In MLU all observations have an “equal weight” during the optimization process. The only way to make some of the observations more or less important is to add or delete selected observations by checking or un-checking their “include” checkboxes.

## Forward Prediction

MLU can be used to predict drawdown at any location or time given a conceptual hydrogeological model with a set of hydraulic properties. This feature can be very useful in designing wells, well fields and aquifer tests or as a teaching aid to allow the student to see the effects of changing parameters on the magnitude and shape of the predicted drawdown curve.

To initiate forward prediction follow these basic steps:

1) Open a new MLU project (Menu bar, File | New or Task bar, New project icon).
2) Complete the “General info” tab as needed.
3) Complete the “Aquifer system” tab as needed (leave all optimization codes blank).
4) Complete the “Pumping wells” tab as needed. At least one pumping well and discharge rate must be entered. The pumping well can also be an observation well.
5) Complete the “Observation wells” tab as needed. At least one observation well with one drawdown measurement must be included. The time-drawdown data point will be displayed in the output graph, so un-check the include box if you don’t want to show these data.
6) Open the “Time graph” tab and click on “Update graph” to display the desired graph.
7) Make changes to the aquifer system, pumping well(s), or observation well(s) input data and repeat Step 6.

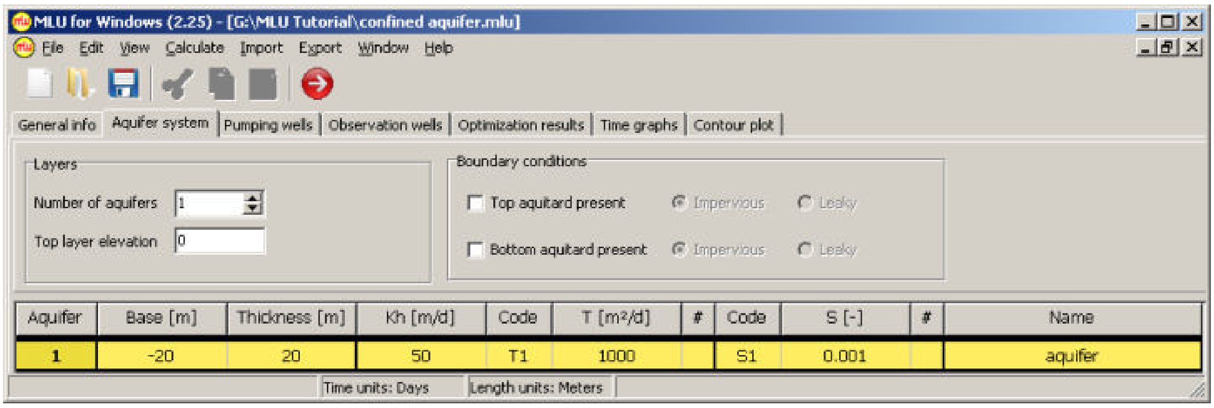
# 4. Conceptualizing Aquifer Systems in MLU

One of the hardest parts of setting up an MLU multi-layer model for aquifer test analysis is vertical discretization. Inexperienced users may think that a complex hydrogeologic system requires many model layers to produce accurate results. However, any model that fits the measurements (or produces the results) accurately enough with a smaller number of layers, can be regarded as a better model. So the art is to keep the model as simple as possible.

This section discusses several common aquifer system types and gives practical advice on how to set them up in MLU.

## Confined aquifer

The simplest aquifer system is a single fully confined aquifer. In MLU, set the number of aquifers to one and remove the top and bottom aquitard by un-checking the corresponding boxes. In the aquifer system table this simple system is identified as a single yellow row between two black bars. The yellow row represents the aquifer and holds the values of its hydraulic properties, while the black bars represent the no-flow boundaries at the top and base.




**Example "Aquifer system" tab for a confined aquifer.**

When starting a new model, the default aquifer thickness is 1 m, the hydraulic conductivity $K_h$ is 1000 m/d and the transmissivity T is 1000 $m^2$/d. The transmissivity is by definition equal to the layer thickness multiplied by $K_h$. T = thickness * $K_h$. The default storativity S is 0.001.

The length units in MLU can be meters (m) or feet (ft). The required unit can be selected after right-clicking the appropriate text on the Status bar (lowest part of the MLU screen). Changing between "m" and "ft" causes MLU to convert all length related input data.

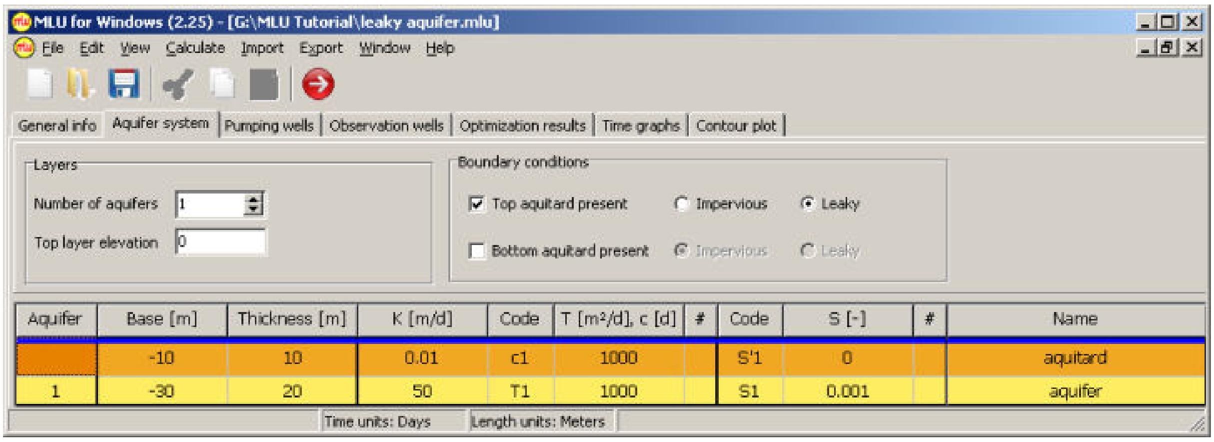
The time units in MLU can be seconds (s), minutes (min), hours (h), days (d) and years (yr). The required unit can be selected after right-clicking the appropriate text on the Status bar. Changing between time units causes MLU to convert all time related values to the new units, as explained in the "Length and time units" section (p. 11).

The elevation of the top of the system above reference level can be set in the input box "Top layer elevation". With an elevation set by the user, the base level of the aquifer is adjusted to any entered positive thickness or the thickness is adjusted to any entered base level provided that it is lower than the elevation of the top.

When the thickness is modified by the user, explicitly or implicitly by altering the base level, $K_h$ is also adjusted in such a way that the transmissivity T remains unaltered.  When $K_h$ is modified by the user, MLU computes T using the layer thickness. If T is modified, $K_h$ is adjusted.

## Leaky aquifer

Let us now assume that there is an aquitard on top of the aquifer, and the aquifer is leaky (semi-confined). To create a leaky aquifer set the number of aquifers to one and check the box "Top aquitard present" and click the "Leaky" button. An orange aquitard row appears above the aquifer and the black bar at the top turns into blue. This blue bar represents a no-drawdown condition at the top of the aquitard, which is the top of the system.




**Example "Aquifer system" tab for a leaky aquifer.**

The labels in the column headers of the aquifer system table depend on the selected row. To select a row, click on any of its cells. A selected cell can be recognized by its bold contents. Note the column heading changes between clicking on an aquifer and aquitard row. In the same columns of the table where an aquifer layer holds the $K_h$, T and S values, an aquitard layer holds $K_v$, c and S' values, representing the hydraulic conductivity for vertical flow ($K_v$), the hydraulic resistance for vertical flow (c) and the storativity of the aquitard (S') respectively.

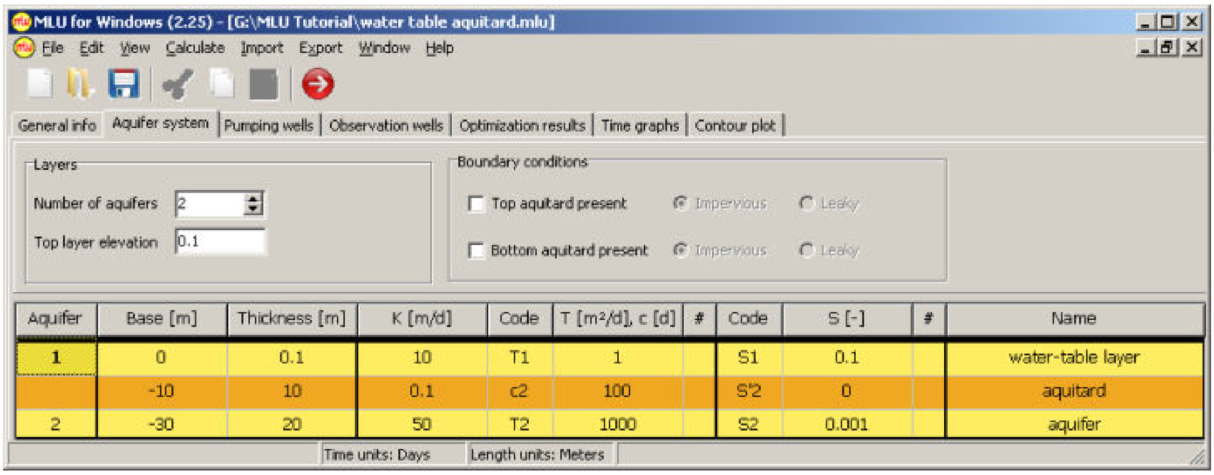
The c-value of a homogeneous aquitard is defined as its thickness divided by the vertical conductivity:  $c = \text{thickness}/K_v$. This hydraulic resistance is the reciprocal of the vertical hydraulic leakance.

Similar to any aquifer, an aquitard also has a thickness and a base level. When the thickness of an aquitard is modified by the user, explicitly or implicitly by altering the base level, $K_v$ is also adjusted in such a way that the c-value remains unaltered.

## Fconductivit

## Leaky aquifer with water-table aquitard

The system becomes more complicated when a water table is present within the aquitard. When the aquifer is pumped, leakage from the aquitard causes the water table to fall, releasing water at the top of the saturated part of the aquitard. When the falling water table does not trigger additional recharge, the system can be considered fully confined. The source of the pumped water is from the elastic storage of the aquifer and from dewatering of the aquitard. So only the storativity of the aquifer and the specific yield of the aquitard are involved.

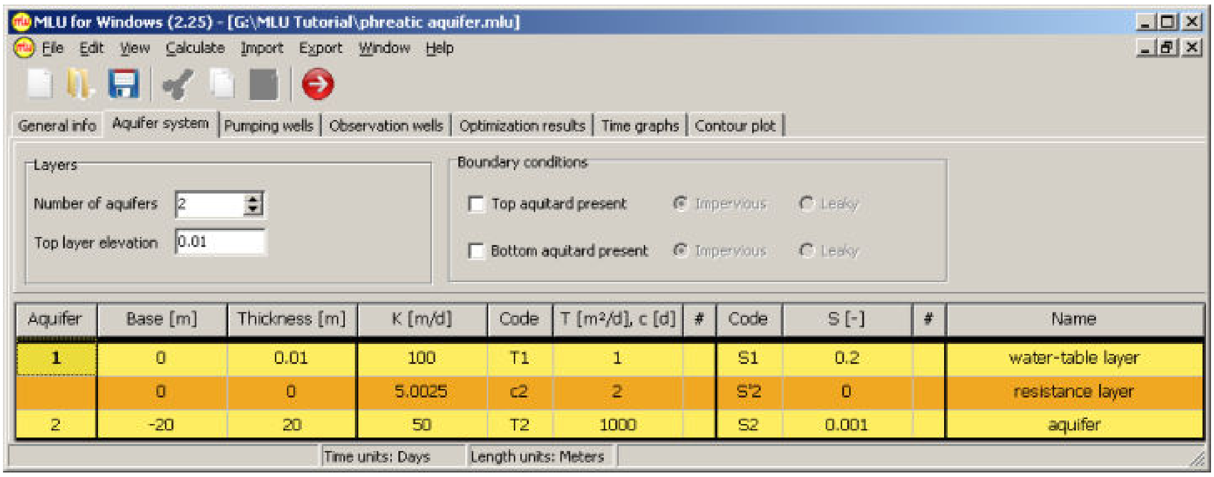



**Example "Aquifer system" tab for a leaky aquifer with water-table aquitard.**

To model this case in MLU we need to add a fictitious "water-table layer" on top of the aquitard to which we can assign a specific yield value. The water-table layer serves as a reservoir that takes up or releases water when the water table rises or falls. No significant horizontal flow component is required and the transmissivity value should be low. Although it will not really affect the model results, one may prefer to use the "aquitard transmissivity" here, defined as the aquitard saturated thickness multiplied by the aquitard horizontal conductivity. The thickness of the water-table layer can be any value, but usually a small value such as 0.1 m may be used.

## Phreatic aquifer

A phreatic aquifer (also water-table aquifer or unconfined aquifer) is characterized by two storage coefficients: the storativity of the aquifer and the specific yield associated with the falling water table in the aquifer. Here too we need an additional “water-table layer” at the top of the system. Because there is no aquitard involved, as discussed in the previous example, the question arises what the hydraulic resistance should be of the resistance layer between the “water-table layer” and the phreatic aquifer. Comparison with other analytical solutions for phreatic aquifers shows that this resistance depends on the assumed conditions near the falling water table, like a delayed or instantaneous drainage from the unsaturated zone. One may assume that this resistance is at least equal to the vertical resistance of the upper third of the phreatic aquifer:  c = aquifer thickness / (3 * aquifer $K_v$). The thickness of the resistance layer can be set to any small value or zero.



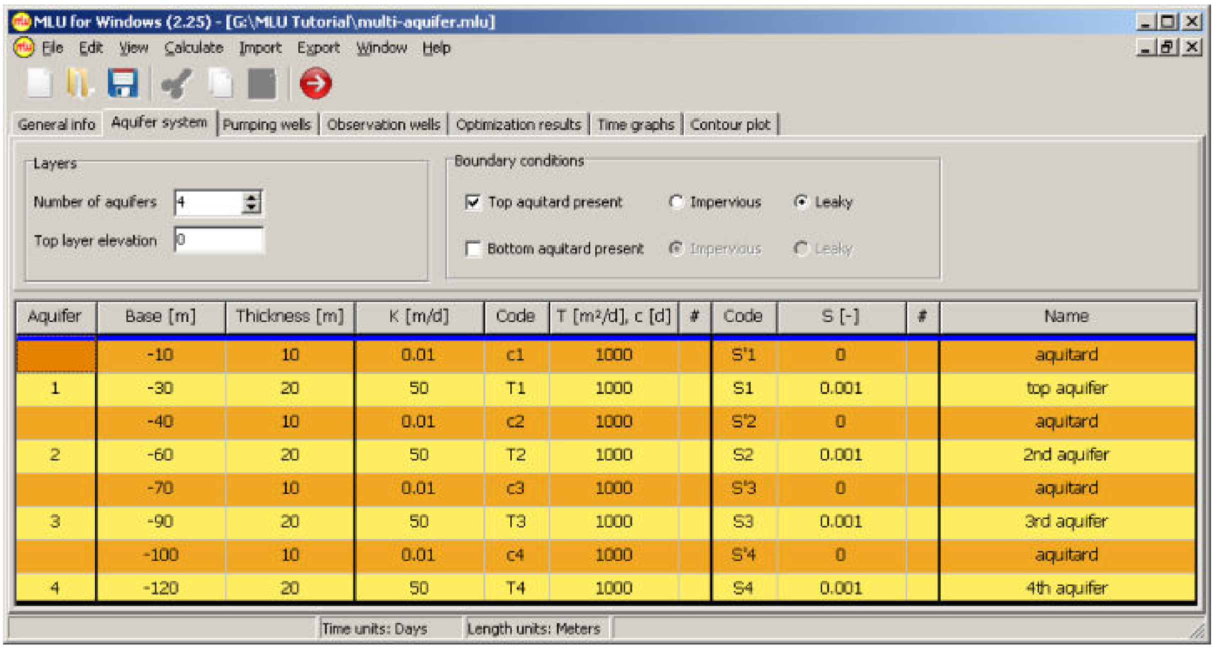

**Example “Aquifer system” tab for a phreatic aquifer.**

More details about phreatic aquifer test analysis are presented in Section 5 – MLU Aquifer Test Examples (Moench, Two-aquifers, Vennebulten).

## Multi-aquifer system

A system that consists of a series of alternating aquifers and aquitards is referred to as a multi-aquifer system. The construction of such a model is straight-forward. It consists of the following steps:

- Select the number of aquifers
- Add or remove aquitards near the top and base of the system
- If top or base aquitards are present, set the boundary conditions
  There are only two options: “impervious” (no-flow) or “leaky” (no-drawdown)
- Enter the “Top layer elevation” and base level or thickness for each aquitard and aquifer
- Enter horizontal conductivity $K_h$ or transmissivity T, and storativity S for each aquifer
- Enter vertical conductivity $K_v$ or resistance c, and storativity S’ for each aquitard
- Enter names for the layers if needed
- Check all input
- Save the project (File | Save or Save icon).




**Example “Aquifer system” tab for a multi-aquifer system.**

The number of aquifers is equal to the number of yellow bars. Only the yellow bars are numbered, starting at the top of the system. Between these aquifers, aquitards are present (orange bars). Aquitards can also form the base and top of the system.

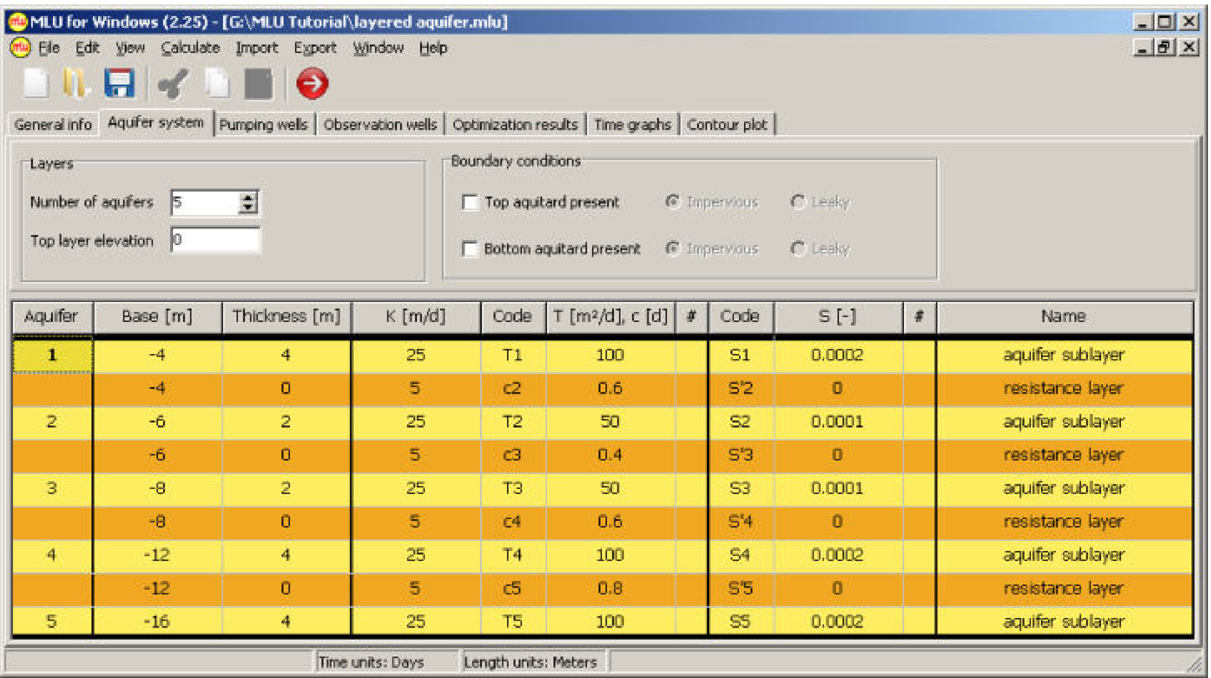
The contents of all cells can be manually modified by the user, except for the first column, which shows the aquifer numbers, and the two code columns. The code columns contain unique codes for all aquifer transmissivities (T1, T2, …) and resistances (c1, c2, …) in one column, and storativities of all aquifers (S1, S2, …) and all aquitards (S’1, S’2, …) in the other. These codes are used at a later stage for automated parameter estimation.

## Layered aquifer

Sometimes it is important to account for the vertical flow component within an aquifer. Vertical flow components within an aquifer may result from:

- Partially penetrating wells
- Layers with different conductivities and/or specific storage (stratified aquifers)
- A rising or falling water table
- Leakage from or into adjacent aquitards.

In such cases the drawdown variation with depth can only be simulated if the aquifer is modeled as a layered aquifer. Vertical flow between adjacent aquifer sublayers depends on the drawdown differences and the vertical resistances between the sublayer centers. The vertical resistance, although not related to an actual aquitard, is also represented by an orange bar in the table. The difference between the resistance layer and an aquitard is that a resistance layer has no thickness. Drawdowns are only computed for the middle of each aquifer sublayer.




| Aquifer | Base [m] | Thickness [m] | K [m/d] | Code | T [m²/d], c [d] | # | Code | S [-] | # | Name |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -4 | 4 | 25 | T1 | 100 | | S1 | 0.0002 | | aquifer sublayer |
| | -4 | 0 | 5 | c2 | 0.6 | | S'2 | 0 | | resistance layer |
| 2 | -6 | 2 | 25 | T2 | 50 | | S2 | 0.0001 | | aquifer sublayer |
| | -6 | 0 | 5 | c3 | 0.4 | | S'3 | 0 | | resistance layer |
| 3 | -8 | 2 | 25 | T3 | 50 | | S3 | 0.0001 | | aquifer sublayer |
| | -8 | 0 | 5 | c4 | 0.6 | | S'4 | 0 | | resistance layer |
| 4 | -12 | 4 | 25 | T4 | 100 | | S4 | 0.0002 | | aquifer sublayer |
| | -12 | 0 | 5 | c5 | 0.8 | | S'5 | 0 | | resistance layer |
| 5 | -16 | 4 | 25 | T5 | 100 | | S5 | 0.0002 | | aquifer sublayer |

**Example "Aquifer system" tab for a layered aquifer.**

In MLU a resistance layer is created by entering a zero-thickness. For these zero-thickness layers the relation between the vertical resistance c and $K_v$ is based on the average thickness of the adjacent aquifer layers:

$$c = (\text{half thickness upper layer} + \text{half thickness lower layer}) / K_v$$

In MLU it is possible to create a complex multi-aquifer system, composed of several stratified aquifers and layered aquitards, however the total number of aquifer layers associated with all aquifers and layered aquitards is limited to a maximum of 80.

# 5. MLU Aquifer Test Examples

Based on the information presented in the preceding sections we will now explore how to use MLU to analyze aquifer tests by discussing the twelve example aquifer test models supplied with MLU. The examples were chosen to represent a range of aquifer types (confined, leaky, phreatic) and test types (pumping, recovery, step-drawdown, slug).

It is assumed that the user is familiar with the MLU User's Guide (Hemker & Post), which contains a complete description of the MLU software including the details of data entry and running the model.

The following table summarizes each example.

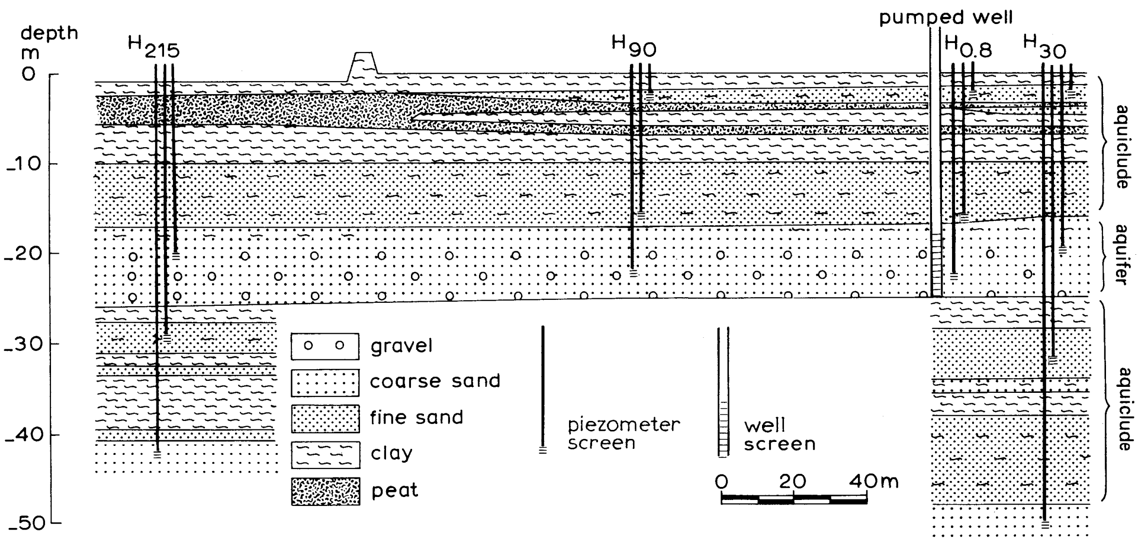
| File name (*.mlu) | System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|---|
| Korendijk | Confined | 1 | 1 | 3 | 2 | T1 S1 |
| Dalem | Leaky | 1 | 1 | 4 | 4 | T1 c1 S1 S'1 |
| Recovery | Leaky | 2 | 1 | 1 | 3 | T1 S1 sk1 |
| Schroth | Confined | 2 | 1 | 3 | 8 | T1 T2 c2 S1 S2 S'2 sk1 rc1 |
| Moench | Phreatic | 6 | 1 | 4 | 0 | verification |
| Two-aquifers | Phreatic | 6 | 1 | 6 | 0 | verification |
| Vennebulten | Phreatic | 3 | 1 | 2 | 4 | T3 c3 S1 S3 |
| Double-porosity | Confined | 2 | 1 | 1 | 3 | T2 c2 S2 |
| Bounded-aquifer | Confined | 1 | 187 | 13 | 0 | verification |
| Step-drawdown | Confined | 2 | 6 | 6 | 7 | T1 sk1 up to sk6 |
| Slugtest | Confined | 1 | 1 | 1 | 2 | T1 S1 |
| Mw-slugtest | Confined | 1 | 1 | 2 | 3 | T1 S1 sk1 |

**Please note that if you are using the lite-version of MLU the results you get may be slightly different from those presented in the following sections because the number of observation wells and aquifers in the lite version example files may have been modified to fit the limitations of the lite program.**

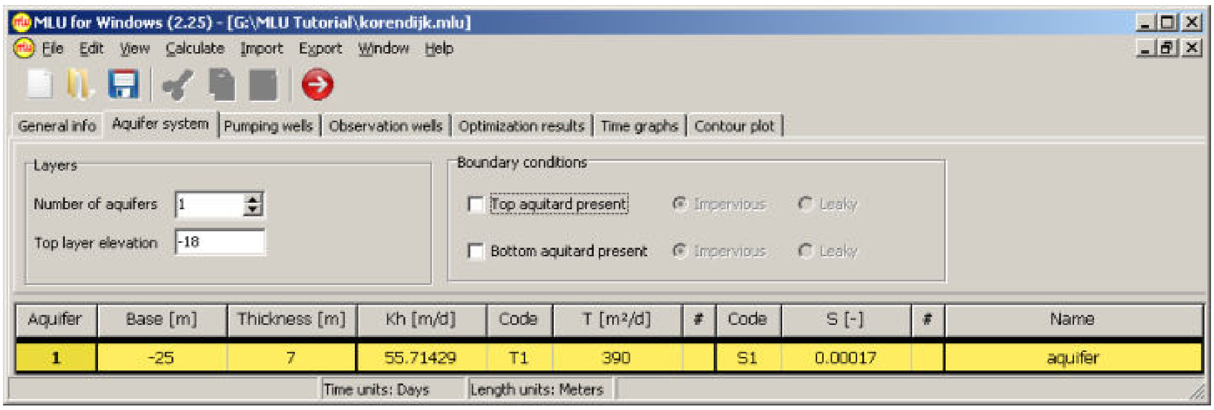
## Korendijk

| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Confined | 1 | 1 | 3 | 2 | T1 S1 |

To demonstrate aquifer test analysis methods in a confined aquifer system K&dR (2000) used a test conducted in the polder Oude Korendijk (Netherlands). They conceptualized the complex stratigraphy shown below as a single confined aquifer. All required data presented in their text and Table 3.1 were entered in MLU including their calculated results (p71, T1 = 390 $m^2$/d and S1 = 1.7x$10^{-4}$) and saved as 'Korendijk.mlu'. The "Aquifer system" tab is shown below.




**Lithological cross-section of the pumping-test site 'Oude Korendijk' (from K&dR, 2000).**




**MLU "Aquifer system" tab – Korendijk (confined aquifer).**

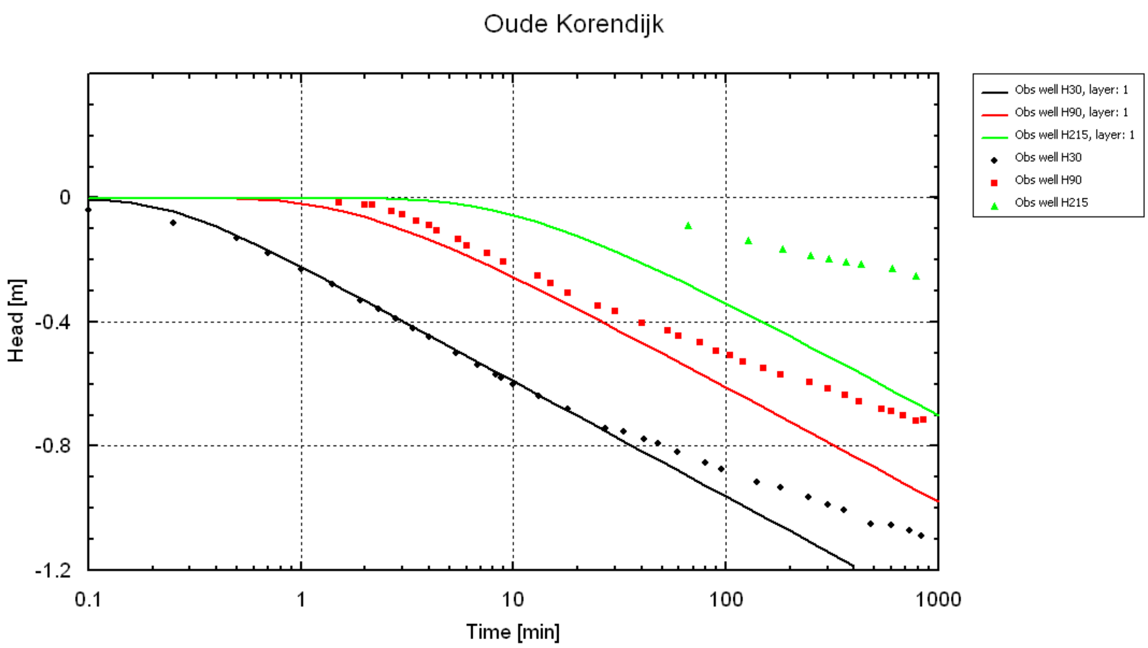
TIP> The positions of the wells are not given, so we accept local coordinates and locate the pumping well at the origin (0,0) and all piezometers along the x-axis. The pumping well and observation well radii are also unknown. In MLU these values are required, so we assume the MLU default values of 0.1 m for the pumping well and 0.02 m for the three observation wells (see "Pumping wells" and "Observation wells" tabs).

TIP> Drawdown observations were taken from K&dR Table 3.1. For each piezometer the first "measurement" is left out. The smallest time that can be entered in MLU is $1 \times 10^{-6}$ days = $1.44 \times 10^{-3}$ minutes. In MLU the drawdown at t = 0 is always zero (initial condition).

All times in Table 3.3 are expressed in minutes, so in MLU the Time units should also be set to 'minutes' in the Status bar at the bottom of the "Aquifer system" tab. Right-click on it and change to minutes in the pop-up window.

TIP> MLU applies the selected time units to the entered data at the time of entry. Changing time units after data entry causes MLU to physically convert the time values to the new units rather than assigning new time units. If you enter the time data with improper time units set, you do not have to re-enter them individually: just copy the block of data (shift + arrow keys to select, ctrl + c to copy), adjust the MLU Time units and paste the data back at the same location (select the upper left cell, ctrl + v to paste). For a more through discussion of time units, see the "Length and time units" section (p. 11).

Now display the time graph ("Time graphs" tab) to see how well the K&dR calculated results (T1 = 390 $m^2/d$ and S1 = $1.7 \times 10^{-4}$) fit the field data (click Update graph). With the X-axis already set to minutes and changing the Y-axis to Linear head (similar to K&dR Figure 3.3) the results are presented below showing that a good fit is only obtained for the first 30 minutes of the nearest piezometer. Run MLU (click the red Optimize icon on the Toolbar) to obtain a sum of squares of 1.6711 $m^2$ for the K&dR results (see "Optimization results" tab).




**Korendijk pumping test: Time-drawdown plot of piezometers $H_{30}$, $H_{90}$ and $H_{215}$.**

Before running MLU to optimize T1 and S1 you must type an ‘a’ and ‘b’ (etc.) into the aquifer system table “# columns” for T1 and S1 to identify these parameter to MLU for optimization (see Users Manual). A solution is found after 4 iterations:

| No. | T 1 | S 1 |
|---|---|---|
| 0 | 390.0 | 1.70E-04 |
| 1 | 452.3 | 2.16E-04 |
| 2 | 449.2 | 2.41E-04 |
| 3 | 445.3 | 2.49E-04 |
| 4 | 444.6 | 2.51E-04 |

The sum of squares is reduced from 1.6711 to 0.6937. However, the fit remains poor, especially for the piezometer at 215 m.

In a fully confined aquifer the drawdown curves for all piezometers always have exactly the same shape: the Theis curve. In the Korendijk example the measured late-time drawdowns increase slower than the computed Theis drawdowns. The late-time data were likely affected by leakage as suggest by K&dR in their summary. Leakage is the subject of the following example (Dalem).

## Dalem

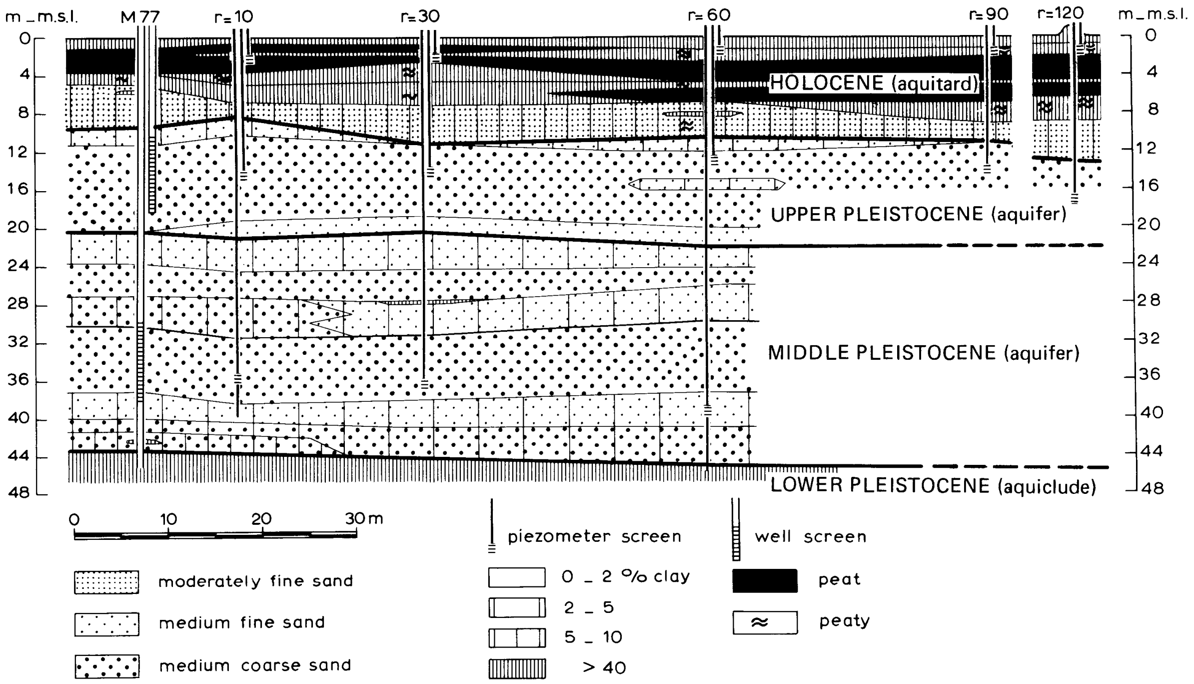
| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Leaky | 1 | 1 | 4 | 4 | T1 c1 S1 S'1 |

Fully confined aquifers don't actually exist in nature. There is always some kind of storage or leakage effect when a confined aquifer is pumped. As explained in Section 3 (p. 9: Storage and leakage), different sources can be identified:

"a" - wellbore storage
"b" - aquifer storage
"c" - aquitard storage
"d" - leakage through aquitards
"e" - water released by a falling water table in an overlying aquitard.

A fully confined aquifer implies that the sources "c", "d" and "e" don't exist. A fully confined aquifer model is acceptable when these sources can be neglected. In the Korendijk example such conditions only lasted for 30 minutes. Later drawdown measurements likely were affected by "c" and "d". In this example we will account for leakage and aquitard storage.

To demonstrate aquifer test analysis methods in a leaky aquifer system K&dR (2000) used a test conducted in Dalem (Netherlands). They conceptualized the complex stratigraphy shown below as a single leaky aquifer. Four piezometers are located 30 to 120 m from the pumping well. The test lasted for eight hours. The diameter of the well is very small, so wellbore storage can be neglected.




**Lithological cross-section of the pumping-test site 'Dalem' (from K&dR, 2000).**

The Dalem aquifer test is a typical example of a leaky aquifer test. The effect of leakage is that the drawdown curves flatten and eventually become constant when steady state is reached. In addition to the transmissivity and storativity, the data allow us to also estimate the hydraulic resistance of the aquitard.

When a leaky aquifer is assumed, aquitard storage is often neglected. Most leaky aquifer analytical solutions assume non-elastic aquitards because this simplifies the mathematics. However, from a soil mechanical point of view there is no reason to disregard aquitard storage.

Leakage and aquitard storage have similar effects on the drawdown when the aquifer is pumped. When the test approaches steady state, leakage increases to its maximum, but the effect of aquitard storage diminishes and eventually disappears. The Dalem test lasted 8 hours and did not reach steady state. Is this period long enough for leakage to have a significant effect on the measured drawdowns? Or is aquitard storage still the dominant vertical flow process? In other words, can we find values for S', for c or for both?

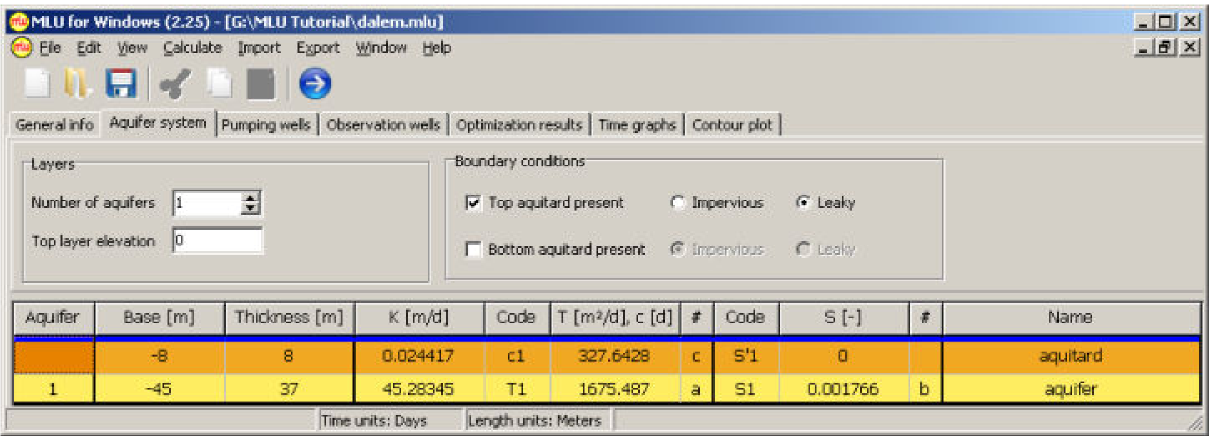
K&dR compared the results of different graphical analytical methods in their Table 4.4 summary. It is clear that most methods assume leakage and neglect aquitard storage (c is calculated, while S' is assumed zero) while two methods take aquitard storage into account.

The time-drawdown data presented in K&dR Table 4.2 were entered and saved as 'Dalem.mlu'. The initial results assume that aquitard storage is negligible (S' = 0).

The MLU results based on linear and log drawdown curve fitting are:

Linear: T = 1676 $m^2$/d (3 %); S = 0.0018 (6%); c = 328 d (22%)
Log: T = 1780 $m^2$/d (3 %); S = 0.0016 (5%); c = 539 d (36%).

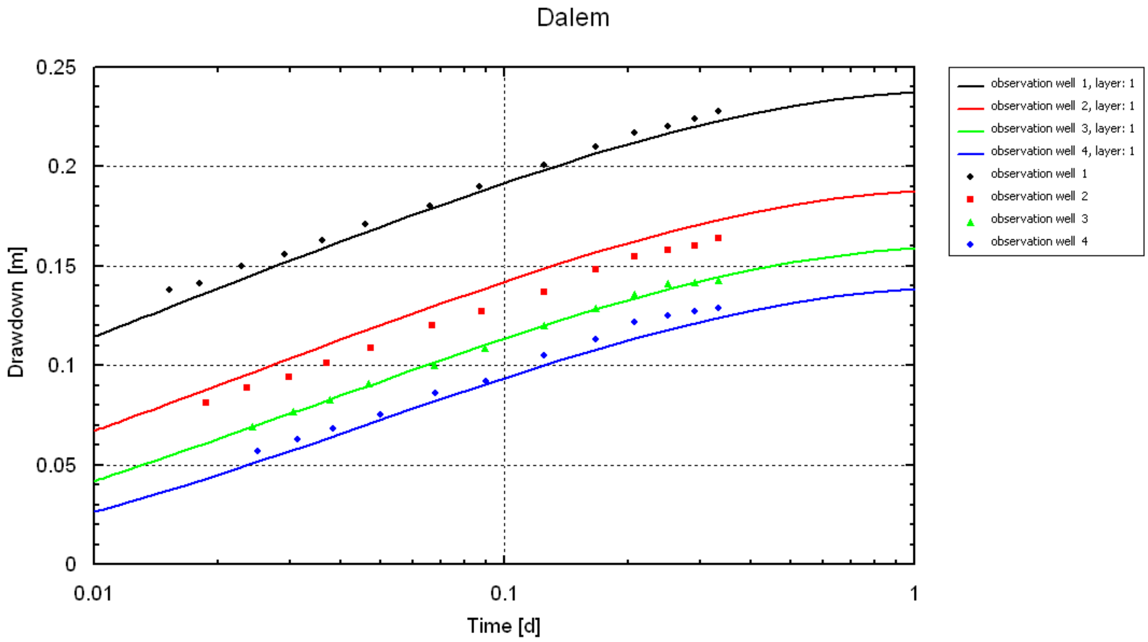
Note: The values in parentheses are the standard deviations expressed as a percentage of the estimated value (see "Optimization results" tab).




**MLU "Aquifer system" tab – Dalem (leaky aquifer with no aquitard storage).**

The available measurements allow estimating the transmissivity and storativity within narrow bounds, but the resistance value is uncertain. Also, the difference between linear and log-transformed least squares solutions is the largest for the resistance values. The MLU results are comparable with the overall K&dR graphical results: T = 1800 $m^2$/d; S = 0.0017; c = 450 d.

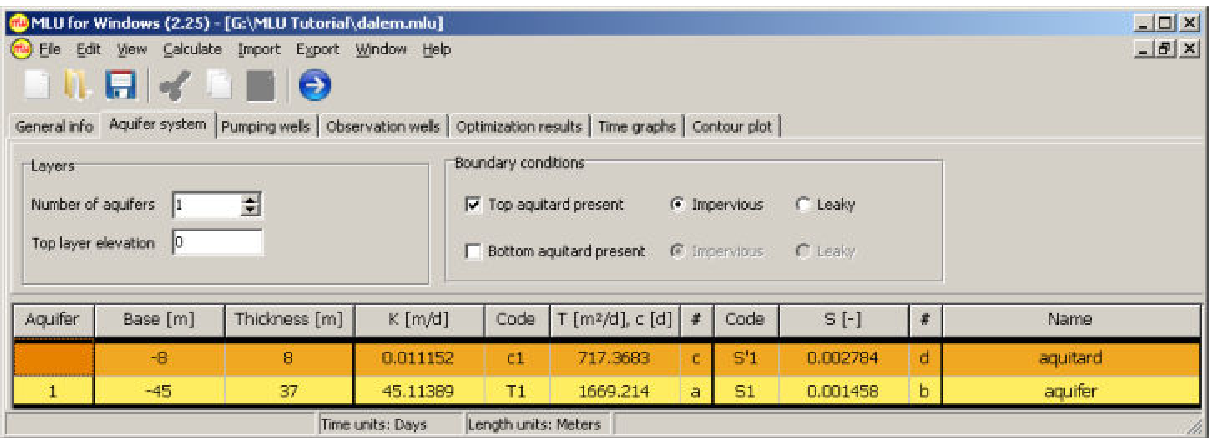
When we look in detail at the linear drawdown curve fitting results, the first impression is that there is a good fit between the model predictions and field measurements. The residual sum of squares is only 0.0018 $m^2$. The differences between the calculated and the measured drawdowns (the residuals) are over 0.01 m for only two measurements, and less than 0.005 m in most cases. However, when we look at them graphically, systematic errors are apparent. For some piezometers systematically too high drawdowns are computed, while for other piezometers the opposite is true.




**Dalem pumping test: Time-drawdown plot of the piezometers at 30, 60, 90 and 120 m from the pumping well.**

These differences are not easy to explain. It seems likely that the stratified character of both the aquifer and of the aquitard, as shown in the lithological cross-section, may play a role.

Now let's check the other extreme: only aquitard storage with no leakage. We ignore leakage in this analysis by selecting an impervious top boundary. We then add aquitard storage by entering an estimated S' of 0.001 and optimize by clicking the red Optimize button. The results are shown below.



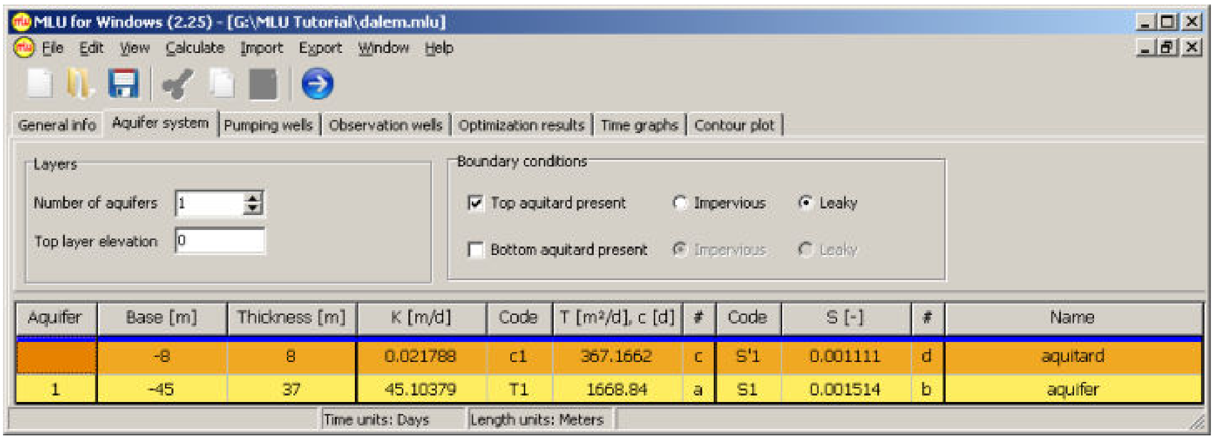

**MLU "Aquifer system" tab – Dalem (with aquitard storage).**

If the previous linear drawdown curve fitting solution is our starting point, we find: T = 1669 (3 %); S = 0.0015 (9%); c = 717 and S' = 0.0028. The residual sum of squares is 0.0017 $m^2$, slightly smaller than the leakage model (see "Optimization results" tab). However, the resistance and the aquitard storativity cannot be obtained with any accuracy. The optimization results also show that the condition number is extremely large (over $10^7$) and that the correlation between c and S' is 100%. This is a clear example of what is called "overparameterization" and "nonuniqueness" of the solution.

In this case, c and S' cannot be computed simultaneously. As explained by K&dR (section 4.2.3) the release of water from aquitard storage depends on the ratio S'/c. When we set c equal to a low constant value of 300 days, we calculate S'= 0.00157. When we change the c to a high constant value of 3000 days, we calculate S' = 0.01163. Obviously the ratio S'/c is somewhere in the range of about $4x10^{-6}$ to $5x10^{-6}$ $day^{-1}$. K&dR (p. 93) find a higher value of S' = $1.1x10^{-5}$, but this value is likely problematic because of the difficulty of obtaining a unique match with one of the Hantush type curves.

K&dR use their S'/c ratio of $1.1x10^{-5}$ $day^{-1}$ together with the obtained c-value to compute an aquitard storativity S' of 0.005. One may wonder if this is a permissible step. One c-value is obtained under the condition that aquitard storage is negligible, the other under the condition that leakage is negligible. Both conceptual models cannot be true at the same time because that would imply a fully confined aquifer and an infinite c-value.

The MLU leaky and aquitard storage results show that the two different hydrogeologic models fit the drawdown data equally well: the model with leakage and no aquitard storage and the model with aquitard storage and no leakage. Therefore, it is likely that both processes play a role. If we choose a leaky top boundary again and optimize all four parameters with linear drawdown curve fitting the results are: T = 1669 (3 %); S = 0.0015 (12 %); c = 367 (41 %) and S' = 0.0011 (111 %). The residual sum of squares is similar to the previous models, so all three conceptual models generate drawdowns that fit the field data about equally well.




**MLU "Aquifer system" tab – Dalem (leaky aquifer with aquitard storage).**

The calculated values are only slightly different than those from the leaky model. The effect of aquitard storage is apparently small and the calculated value for S' remains uncertain. A longer duration aquifer test would likely have helped estimate the c-value with more accuracy. The user is encouraged to take the lessons learned from this leaky aquifer example and experiment with the previous Korendijk example by converting it to a leaky aquifer conceptual model.

## Recovery

| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Leaky | 2 | 1 | 1 | 3 | T1 S1 sk1 |

In 1981 recovery tests were conducted on each of five pumping wells at a water supply pumping station near the village of Hardinxveld-Giessendam (Netherlands). The pumping station is located on the North bank of the river Waal, 10 kilometers downstream of Dalem. The purpose of the tests was to assess the local variation in transmissivity and to quantify the head loss of each well caused by clogging. There were no observation wells or piezometers in the well field.

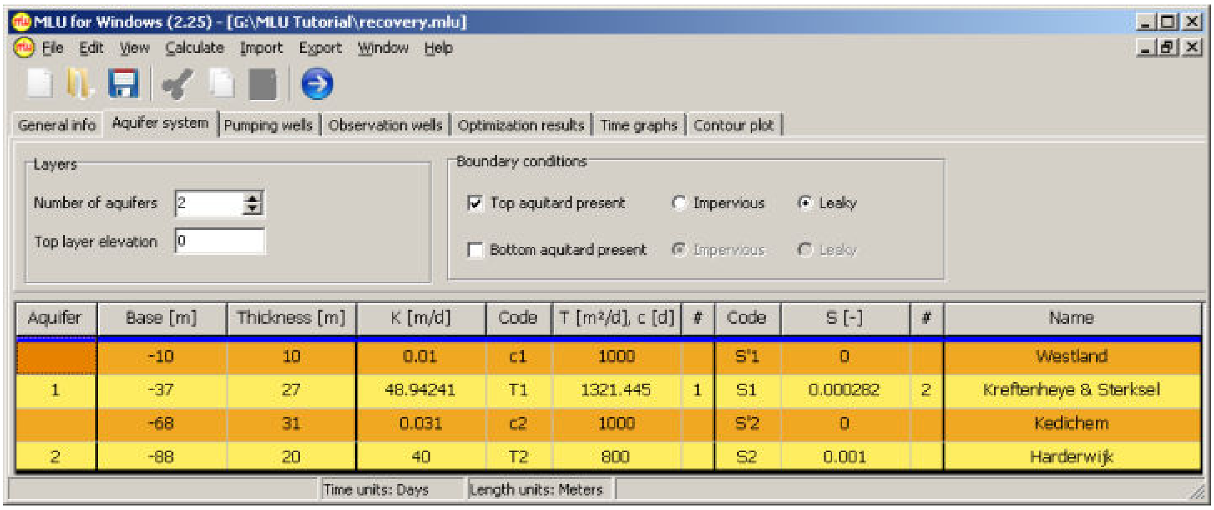
The wells are screened in the upper aquifer, 10 to 37 m below ground surface. The stratigraphy is similar to that of Dalem, but regional borehole information shows that a second aquifer is present at a depth of 68 to 88 m. Although little information about the deeper layers is available, they are included in the aquifer system because of the principle that including guessed properties of little-known layers is preferable to disregarding them altogether. The entered values for c1 and c2 (1000 d), T2 (800 $m^2$/d) and S2 (0.001) are rough estimates, based on regional pumping test information. Note: sensitivity testing will be conducted later to evaluate the influence of these deep aquifer values on the overall optimization.

The well for this example was pumped at 77 $m^3$/h (1848 $m^3$/d) for 20 minutes. The drawdown was measured in the pumped well casing during pumping and for 30 minutes afterwards.

Two hydraulic properties were optimized (T1 and S1) as well as the pumping well skin factor (see below and “Pumping wells” tab for skin).

Linear drawdown curve fitting: T = 1391 $m^2$/d (2 %), S = 0.0008 (91 %), Skin = 7.3 (9 %).
Log drawdown curve fitting: T = 1321 $m^2$/d (1 %), S = 0.00028 (13 %), Skin = 6.1 (4 %).

Note: The values in parentheses are the standard deviations expressed as a percentage of the estimated value (see “Optimization results” tab).

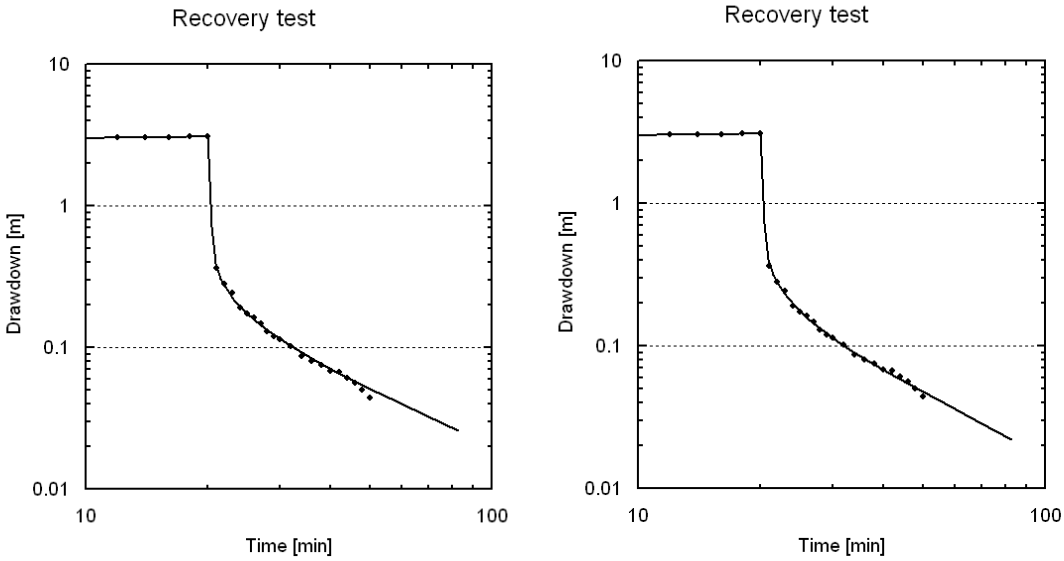



**MLU "Aquifer system" tab – Recovery (two layer model with logarithmic drawdown curve fitting).**

TIP> To help decide which curve fitting solution should be selected as the most appropriate, a comparison of the residual sums of squares is of little help. The linear solution always produces a sum of squares that is equal to or less than the log solution. In the present case the sums of squares are 0.0020 $m^2$ for the linear solution and 0.0025 $m^2$ for the log solution.

The reasoning supporting the choice of the log drawdown curve fitting solution is as follows. Typically a recovery test shows a large difference between the large drawdowns during pumping and the small drawdowns during recovery. The accuracy of the measured drawdowns during pumping is probably significantly less than the drawdowns during recovery because the discharge rate of the pump is likely to vary, especially at the start of the test. Also, the drawdowns were measured by hand and accurately measuring the depth of a shallow water level is easier than a deeper one.

The preference for log drawdown curve fitting is also based on comparison of the results. Although both curve fitting solutions fit the data very well, the log curve fitting solution fits the recovery data slightly better. Visual inspection of both curves on a log drawdown scale and an interval of 10 to 100 minutes shows that the MLU generated drawdown curve based on the log curve fitting solution matches most of the recovery measurements, but the linear curve fitting solution curve is generally a little too low during the start of the recovery and generally a little too high during the last part. These differences can also be checked in the calculated minus observed drawdowns column (Cal-Obs) in the "Optimization results" tab.




**Hardinxveld-Giessendam recovery test: Slight difference between linear (left) and log (right) drawdown curve fitting solutions.**

It is easy to check the influence of including the rough estimates of the fixed parameters for the deeper aquifer system. If the T2, S2, c2 and S'2 values are changed within reasonable site specific ranges (i.e., sensitivity testing) the primary parameters T1 and S1 only change minimally.

Using the calculated values of the skin factor and transmissivity we estimate from this recovery test that the clogging of the well screen caused a head loss of 1.35 m, using Eq. 14.23 of K&dR. For a definition and discussion of the skin factor see Chapter 14 of K&dR.

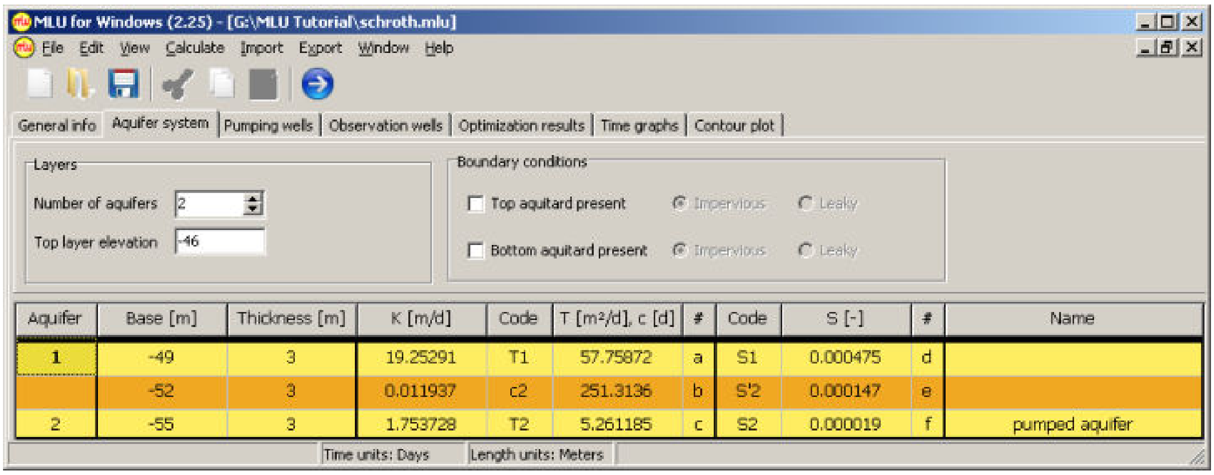
## Schroth

| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Confined | 2 | 1 | 3 | 8 | T1 T2 c2 S1 S2 S'2 sk1 rc1 |

Unfortunately only a few published aquifer tests deal with multi-layered aquifer systems. One excellent example is an aquifer test conducted near San Francisco in 1993 and published in Ground Water by Schroth & Narasimhan (S&N) in 1997. Although the authors speak of a "leaky aquifer test" the test is what can be referred to as a "fully confined two-aquifer system". The lower aquifer is pumped while drawdowns are measured in the pumping well and in two observation wells, one in the upper and one in the lower aquifer.

S&N analyzed their data by first using the conventional type-curve matching technique for the pumped aquifer wells. Then they used a finite-difference model for radial flow to include the upper aquifer. They searched for the six unknown layer parameters by trial and error, until an acceptable match was found. Attention was also paid to the conductivity and storativity of the pumping well skin. With a total of eight unknown parameters, finding a "best fit" solution was likely time-consuming.

To analyze the drawdown data with MLU, 50 drawdowns were selected from the published graph (S&N, Figure 4). Because the drawdowns in the pumped well are much larger (factor 10 or more) than in the two observation wells and the pumped well drawdowns are likely affected by pumping rate variations, we used the log drawdown curve fitting technique. The optimized results are shown below.




**MLU Aquifer system tab – Schroth (two-layer model with logarithmic curve fitting).**

In the following table the main results of MLU and S&N are summarized.

**Comparison of MLU and S&N results.**

| **Parameter** | **MLU** | **Parameter** | **MLU** | | **Schroth & Narasimhan** | |
|---|---|---|---|---|---|---|
| | best estimate | | best estimate (value*$10^{-5}$) | σ [a] (%) | best estimate (value*$10^{-5}$) | range (value*$10^{-5}$) |
| T1 ($m^2$/d) | 58 | K1 (m/s) | 22 | 8 | 20 | 15 - 32 |
| S1 | 4.7 $10^{-4}$ | Ss1 | 16 | 15 | 12 | 8.2 - 15 |
| c2 (day) | 251 | K' (m/s) | 0.014 | 3 | 0.01 | 0.009 - 0.014 |
| S'2 | 1.5 $10^{-5}$ | Ss'2 | 5 | 9 | 3 | 0.9 - 5 |
| T2 ($m^2$/d) | 5.3 | K2 (m/s) | 2.0 | 2 | 4 | 3 - 5 |
| S2 | 1.9 $10^{-5}$ | Ss2 | 0.6 | 7 | 1.5 | 1.2 - 2.2 |

[a] MLU calculated standard deviations expressed as a percentage of the estimated parameter value for optimization without skin factor.

The first two columns present the MLU results when the hydraulic properties of all three layers are optimized. In the third and fourth column these values are converted to the properties as presented by S&N. Ss is the specific storage. All layer thicknesses are assumed to be 3 meters. The last two columns show the numerical results as presented in Table 2 of S&N.

Comparison of both solutions shows that there are large differences, especially for the pumped aquifer where the MLU estimated values of transmissivity and the storativity are at least a factor two lower than estimated by S&N.

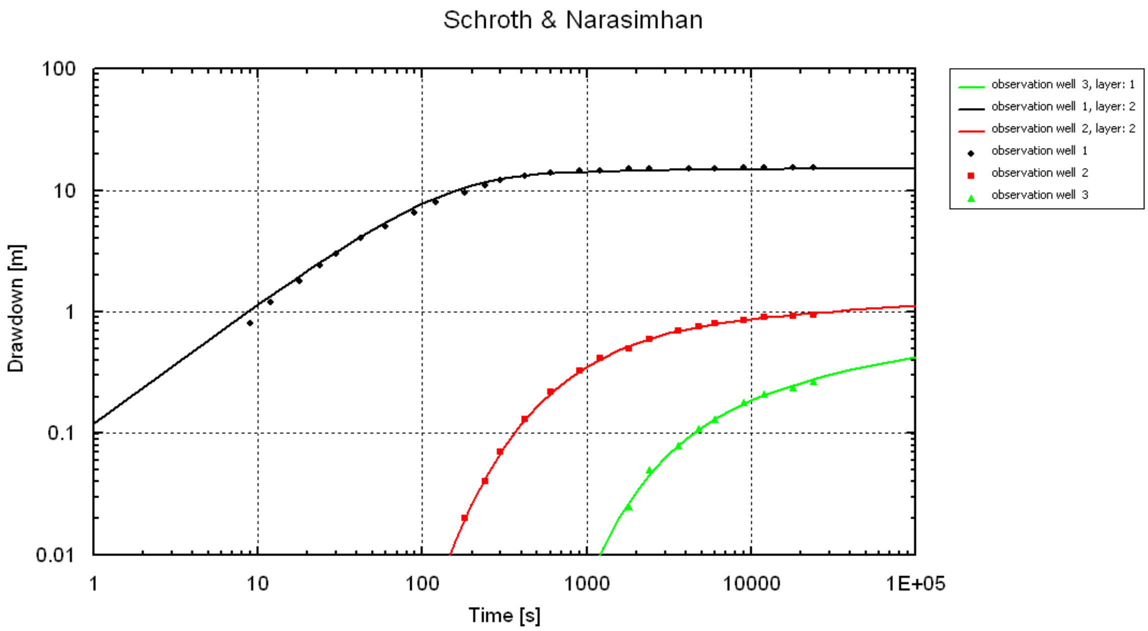
In addition to the above six parameters S&N compute the conductivity and specific storage of a 0.025 m thick skin around the well screen gravel pack. The skin conductivity is related to the skin factor in the following way: skin factor = (Kr * ds)/(Ks * rw), where Kr is the radial conductivity of the screened aquifer, ds is the skin thickness, Ks is the skin conductivity and rw is the well radius. Using the S&N estimated values this results in a skin factor of 4 to 7.

When the skin factor is added to the parameters to be optimized in MLU, it cannot be determined with any accuracy. The correlation between T2 and the skin factor is 100%. With an unknown skin factor, the best estimates for the six layer properties remain basically unaltered, but in a wider range.

To reproduce the solution found by S&N, the transmissivity of the pumped aquifer was set to a value of 10.368 $m^2$/d, which corresponds with their conductivity value of 4 m/s. With the transmissivity fixed, a good match can be obtained. In this analysis the skin factor also can be optimized and was estimated to be 5.2. Most parameters are still outside the ranges given by S&N. In this suboptimal solution the sum of squares is 5.5 $m^2$, while it is 2.2 $m^2$ for the optimal solution.

**Results of successive iterations while changing from the suboptimal to the optimal solution.**

| No. | T1 | T2 | c2 | S1 | S2 | S'2 | Sk1 |
|---|---|---|---|---|---|---|---|
| 0 | 55.3 | 10.4 | 368.8 | 0.000165 | 0.0000453 | 0.000130 | 5.20000 |
| 1 | 55.5 | 10.3 | 367.6 | 0.000166 | 0.0000450 | 0.000130 | 5.10000 |
| 2 | 55.9 | 10.2 | 364.1 | 0.000167 | 0.0000443 | 0.000132 | 5.00000 |
| 3 | 56.7 | 9.9 | 357.3 | 0.000170 | 0.0000431 | 0.000136 | 4.70000 |
| 4 | 57.8 | 9.5 | 345.3 | 0.000178 | 0.0000408 | 0.000142 | 4.30000 |
| 5 | 59.2 | 8.8 | 325.9 | 0.000199 | 0.0000372 | 0.000149 | 3.50000 |
| 6 | 60.2 | 7.8 | 298.9 | 0.000250 | 0.0000319 | 0.000157 | 2.40000 |
| 7 | 59.9 | 6.5 | 269.0 | 0.000355 | 0.0000253 | 0.000158 | 1.10000 |
| 8 | 58.6 | 5.5 | 253.5 | 0.000450 | 0.0000203 | 0.000152 | 0.18800 |
| 9 | 57.9 | 5.4 | 252.8 | 0.000463 | 0.0000194 | 0.000148 | 0.10300 |
| 10 | 57.8 | 5.3 | 252.0 | 0.000469 | 0.0000192 | 0.000147 | 0.05120 |
| 11 | 57.8 | 5.3 | 251.7 | 0.000472 | 0.0000190 | 0.000147 | 0.02520 |
| 12 | 57.8 | 5.3 | 251.5 | 0.000474 | 0.0000190 | 0.000147 | 0.01270 |
| 13 | 57.8 | 5.3 | 251.4 | 0.000474 | 0.0000189 | 0.000147 | 0.00638 |
| 14 | 57.8 | 5.3 | 251.3 | 0.000475 | 0.0000189 | 0.000147 | 0.00298 |
| 15 | 57.8 | 5.3 | 251.3 | 0.000475 | 0.0000189 | 0.000147 | 0.00161 |
| 16 | 57.8 | 5.3 | 251.3 | 0.000475 | 0.0000189 | 0.000147 | 0.00085 |




**Results of log drawdown curve fitting of the Schroth & Narasimhan data.**

It is unclear why S&N used the value of $4 \times 10^{-5}$ m/s for the pumped aquifer conductivity. When their initial type-curve matching technique for the pumped aquifer is reconstructed with MLU, only the early time measurements can be matched. The resulting conductivity is $3.6 \times 10^{-5}$ m/s. It is possible that their initial interpretation with Theis curves put them a little off track.

Inspection of the MLU optimization results with six estimated parameters shows that the pumping well curve has systematic differences between the calculated and observed drawdowns. During the first five minutes all calculated drawdowns are too high. After that time they are too low. This may be caused by a variable pumping rate at very early times.

A better fit can be obtained when the well casing radius is also estimated. This results in a slightly increased radius (0.053 m), while other parameters remain practically the same. The sum of squares reduces from 2.2 $m^2$ to 0.6 $m^2$. However, S&N mention plumbing inside the wellbore casing and so a smaller effective well casing radius than stated is more likely than a larger one. Full details of the results of this optimization run are given in the MLU User's Guide.

After pumping for six hours the pump was stopped and recovery was monitored for two hours. The recovery data could help give a better estimate of T2 and the skin factor, but these data are not presented in the S&N publication.

## Moench

| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Phreatic | 6 | 1 | 4 | 0 | verification |

Moench (1997) developed an analytical solution for flow to a partially penetrating, finite-diameter well in a homogeneous, vertically anisotropic water-table aquifer. To verify the analytical solution a finite difference model was used to compute drawdowns in a partially-penetrating pumping well and four piezometers of a hypothetical aquifer. Subsequently, the simulated data were used to estimate the hydraulic properties. An almost identical hypothetical model was published two years later in 1999 by Barlow & Moench (B&M) and the analytically derived drawdowns presented in this latter report are used here. The purpose of this MLU example is to check how well MLU can simulate drawdowns in an vertically anisotropic water-table aquifer.

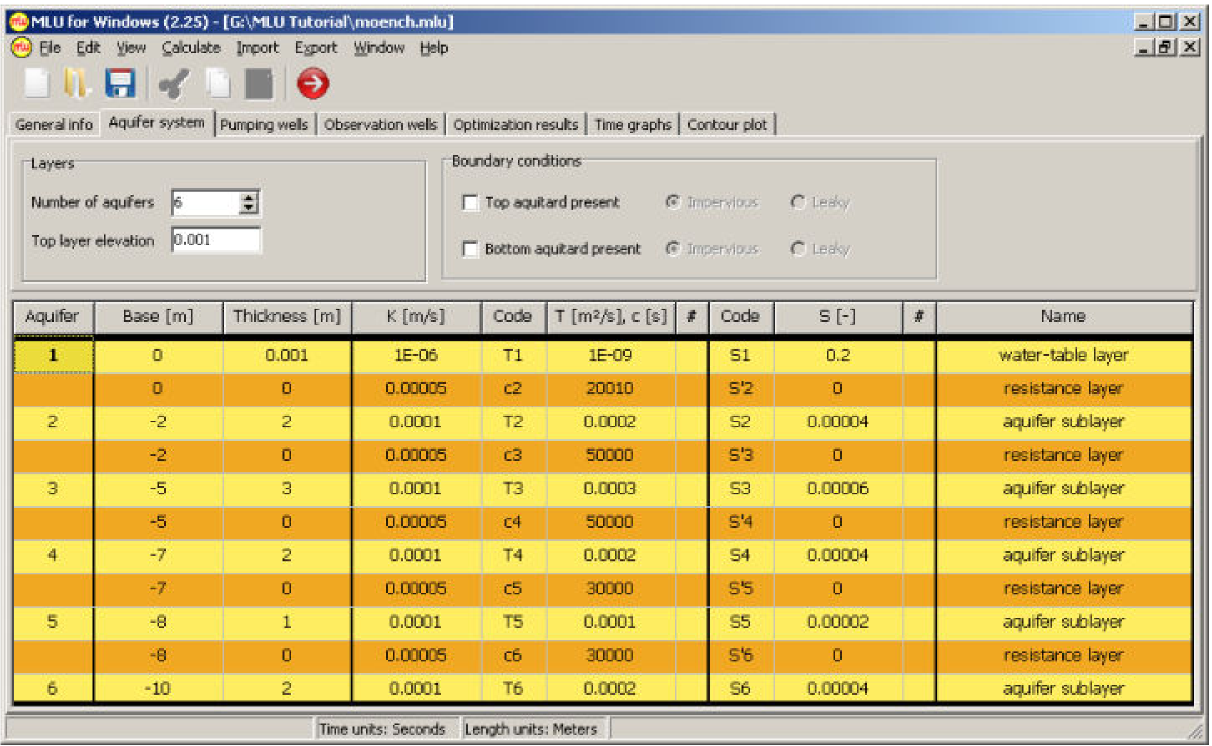
Although the derived B&M well flow solution is for a single homogeneous aquifer, the MLU model requires several layers for two reasons: 1) because of the water-table condition and 2) because of the vertical flow components associated with the partially-penetrating pumping well. Another and generally applicable reason to add sublayers in the case of a homogeneous leaky or phreatic aquifer is that the more sublayers created, the more accurately the vertical flow components can be simulated.

The saturated thickness of the aquifer is 10 m, while the pumping well is only screened in the lower half: 5 to 10 m deep. The four piezometers are located at two radial distances (3.16 and 31.6 m) and at two depths (1 and 7.5 m). To keep the piezometers in the middle of a sublayer and to account for the screened interval of the pumping well, the simplest subdivision of the aquifer thickness would be to choose layers of 2, 3, and 5 m thick from top to bottom. To increase the accuracy of the vertical flow component simulation in the lower part of the aquifer, we added two more layers resulting in layer thicknesses of 2, 3, 2, 1, and 2 m. This is a subjective decision, balancing accuracy and practicability. An alternative subdivision of the aquifer is: 3 layers of 2/3 m thick (to keep the shallow piezometers in the middle of a layer) and 8 layers of 1 m thick. All resistance layers have a zero thickness.

As in all systems with a falling water table, we also need an additional layer at the top of the system to simulate the dewatering effects near a falling water table. The thickness and the conductivity of this layer are set to very low values of 0.001 m and $1x10^{-6}$ m/d respectively, to impede horizontal flow in this layer. Such low values are not really required: we could also have chosen 0.1 m and $1x10^{-4}$ m/d as if this is the top most layer of the aquifer. However, the thickness of this slightly thicker top layer should then be subtracted from that of the next lower layer. A specific yield of 0.2 is assigned as the storage coefficient of this water-table layer.

The other hydraulic properties, as presented in Table 3 of B&M, are the horizontal and vertical conductivity ($1x10^{-4}$ and $5x10^{-5}$ m/d, respectively), and the specific storage ($2x10^{-5}$). The horizontal and vertical conductivities were directly entered in the aquifer system table to generate the layer transmissivities and resistances. The B&M specific storages had to be multiplied by the layer thicknesses to calculate the layer storativities for entry into MLU.

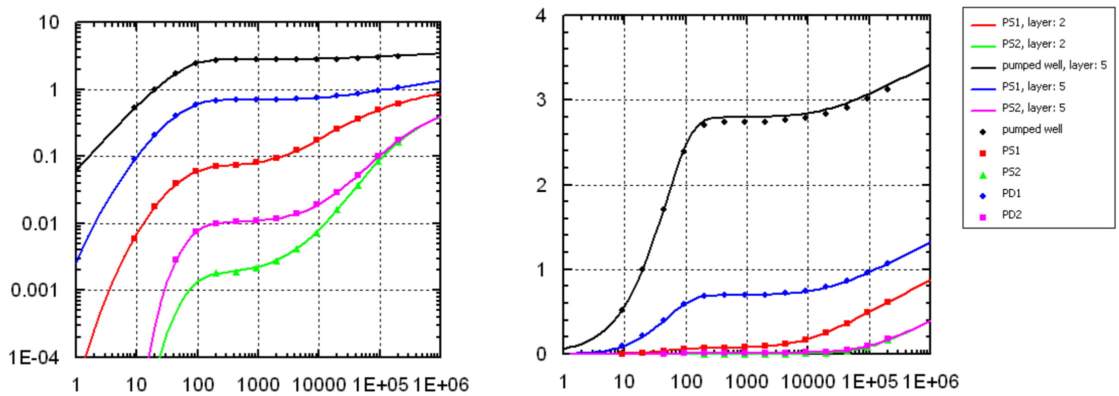
The fully populated MLU "Aquifer system" tab is shown below.




| Aquifer | Base [m] | Thickness [m] | K [m/s] | Code | T [m²/s], c [s] | # | Code | S [-] | # | Name |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0.001 | 1E-06 | T1 | 1E-09 | | S1 | 0.2 | | water-table layer |
| | 0 | 0 | 0.00005 | c2 | 20010 | | S'2 | 0 | | resistance layer |
| 2 | -2 | 2 | 0.0001 | T2 | 0.0002 | | S2 | 0.00004 | | aquifer sublayer |
| | -2 | 0 | 0.00005 | c3 | 50000 | | S'3 | 0 | | resistance layer |
| 3 | -5 | 3 | 0.0001 | T3 | 0.0003 | | S3 | 0.00006 | | aquifer sublayer |
| | -5 | 0 | 0.00005 | c4 | 50000 | | S'4 | 0 | | resistance layer |
| 4 | -7 | 2 | 0.0001 | T4 | 0.0002 | | S4 | 0.00004 | | aquifer sublayer |
| | -7 | 0 | 0.00005 | c5 | 30000 | | S'5 | 0 | | resistance layer |
| 5 | -8 | 1 | 0.0001 | T5 | 0.0001 | | S5 | 0.00002 | | aquifer sublayer |
| | -8 | 0 | 0.00005 | c6 | 30000 | | S'6 | 0 | | resistance layer |
| 6 | -10 | 2 | 0.0001 | T6 | 0.0002 | | S6 | 0.00004 | | aquifer sublayer |

**MLU "Aquifer system" tab –Moench (six layer phreatic model).**

The pumping well discharge (2x10$^{-3}$ m$^3$/s), radii (0.1 and 0.1 m) and skin factor (0) were entered in the pumping wells table. The well is screened in layers 4, 5, and 6. The shallow and deep piezometers are located in layers 2 and 5. The 14 times (in seconds) and drawdowns (in m) for each of the four observation wells are taken from B&M Table 5.



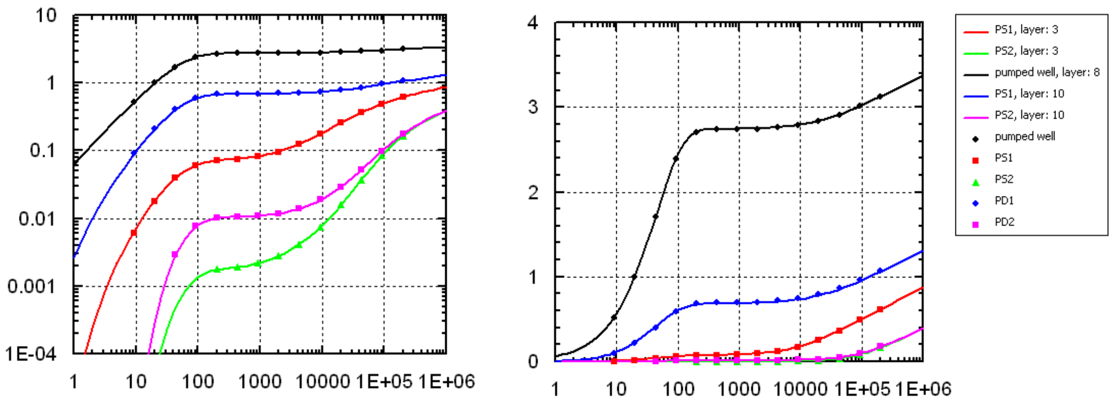

**Barlow & Moench drawdown results (dots) compared to the six-layer MLU results (curves).**

On a log drawdown scale the MLU computed drawdown curves fit the B&M model data well. However, on a linear drawdown scale the MLU drawdowns are up to 2% higher than obtained by the Moench solution. To help determine if the multi-layer approach causes this difference, a 12-layer model was set up, with all layers 1 m thick or less. The 12-layer MLU computed drawdowns more accurately reproduced the B&M results (see below).

| Aquifer | Base [m] | Thickness [m] | K [m/s] | Code | T [m²/s], c [s] | # | Code | S [-] | # | Name |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0.001 | 1E-06 | T1 | 1E-09 | | S1 | 0.2 | | water-table layer |
| | 0 | 0 | 0.00005 | c2 | 7000 | | S'2 | 0 | | resistance layer |
| 2 | -0.7 | 0.7 | 0.0001 | T2 | 0.00007 | | S2 | 7E-06 | | aquifer sublayer |
| | -0.7 | 0 | 0.00005 | c3 | 13000 | | S'3 | 0 | | resistance layer |
| 3 | -1.3 | 0.6 | 0.0001 | T3 | 0.00006 | | S3 | 6E-06 | | aquifer sublayer |
| | -1.3 | 0 | 0.00005 | c4 | 13000 | | S'4 | 0 | | resistance layer |
| 4 | -2 | 0.7 | 0.0001 | T4 | 0.00007 | | S4 | 7E-06 | | aquifer sublayer |
| | -2 | 0 | 0.00005 | c5 | 17000 | | S'5 | 0 | | resistance layer |
| 5 | -3 | 1 | 0.0001 | T5 | 0.0001 | | S5 | 0.00001 | | aquifer sublayer |
| | -3 | 0 | 0.00005 | c6 | 20000 | | S'6 | 0 | | resistance layer |
| 6 | -4 | 1 | 0.0001 | T6 | 0.0001 | | S6 | 0.00001 | | aquifer sublayer |
| | -4 | 0 | 0.00005 | c7 | 20000 | | S'7 | 0 | | resistance layer |
| 7 | -5 | 1 | 0.0001 | T7 | 0.0001 | | S7 | 0.00001 | | aquifer sublayer |
| | -5 | 0 | 0.00005 | c8 | 20000 | | S'8 | 0 | | resistance layer |
| 8 | -6 | 1 | 0.0001 | T8 | 0.0001 | | S8 | 0.00001 | | aquifer sublayer |
| | -6 | 0 | 0.00005 | c9 | 20000 | | S'9 | 0 | | resistance layer |
| 9 | -7 | 1 | 0.0001 | T9 | 0.0001 | | S9 | 0.00001 | | aquifer sublayer |
| | -7 | 0 | 0.00005 | c10 | 20000 | | S'10 | 0 | | resistance layer |
| 10 | -8 | 1 | 0.0001 | T10 | 0.0001 | | S10 | 0.00001 | | aquifer sublayer |
| | -8 | 0 | 0.00005 | c11 | 20000 | | S'11 | 0 | | resistance layer |
| 11 | -9 | 1 | 0.0001 | T11 | 0.0001 | | S11 | 0.00001 | | aquifer sublayer |
| | -9 | 0 | 0.00005 | c12 | 20000 | | S'12 | 0 | | resistance layer |
| 12 | -10 | 1 | 0.0001 | T12 | 0.0001 | | S12 | 0.00001 | | aquifer sublayer |

Time units: Seconds Length units: Meters

**MLU "Aquifer system" tab –Moench (12-layer phreatic model).**




**Barlow & Moench drawdown results (dots) compared to the 12-layer MLU results (curves).**

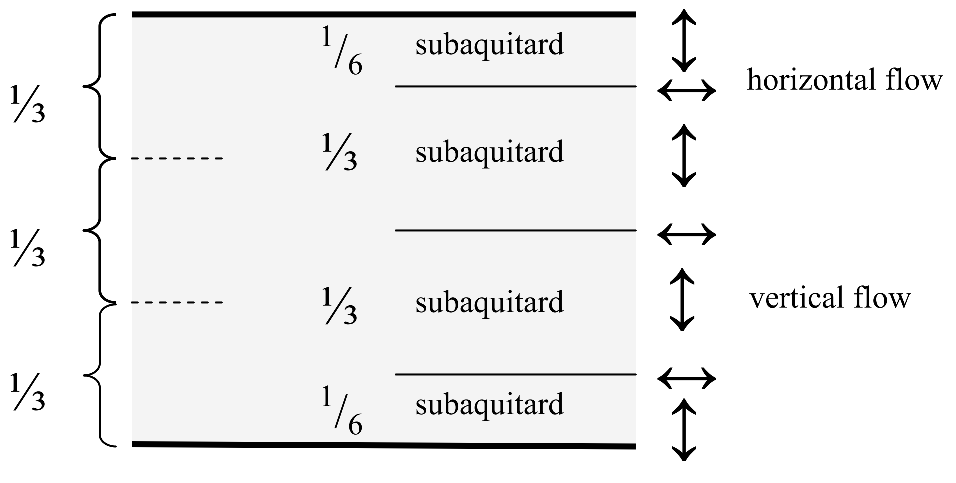
The time-drawdown curves show a typical S-shape, with a flat intermediate-time segment. This is a characteristic feature of drawdown curves for aquifer systems with a falling water table, i.e., phreatic aquifers or water-table aquitards. It is called the delayed water-table response and it is caused by the difference in the values of the elastic storage coefficient (storativity) and specific yield, which differ by orders of magnitude.

## Two-aquifers

| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Phreatic | 6 | 1 | 6 | 0 | verification |

Two-aquifers is another synthetic aquifer analysis problem. It was used by Lebbe & De Breuck (L&dB) in 1995 to validate their finite-difference model for radial flow in layered aquifer systems. Two homogeneous aquifers are separated by a homogeneous aquitard. The upper aquifer is phreatic. The aquifers and the aquitard are each 10 m thick. The aquitard is anisotropic: the horizontal and vertical conductivities are 0.5 and 0.1 m/d respectively.

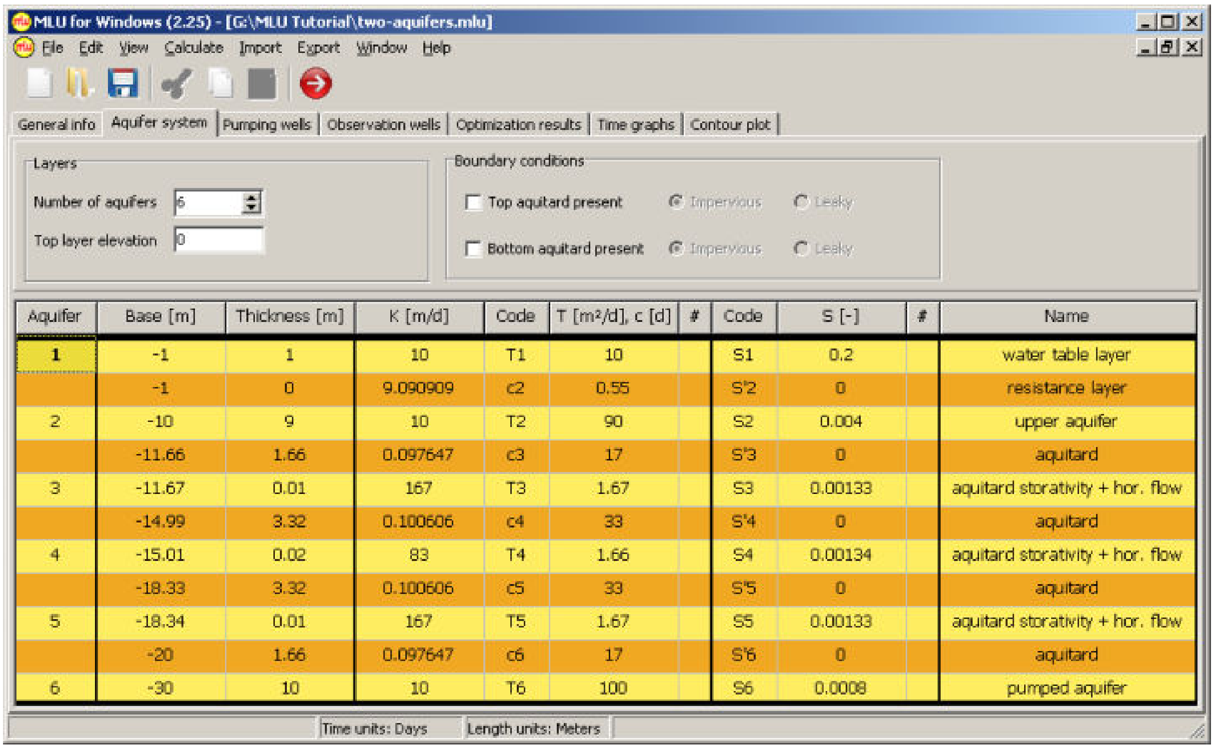
L&dB split their aquitard into three layers with horizontal flow and a storage coefficient, separated by zero-thickness resistance layers, to simulate the diffusive cross flow into the lower aquifer. Diffusive cross flow, as opposed to linear cross flow, is characterized by a depth-variant flux distribution caused by the changing aquitard storage. Linear cross flow occurs when aquitard storage is disregarded; it is depth-invariant and proportional to the head-difference over the aquitard. Although diffusive cross flow can be modeled by a single aquitard in MLU, it is the horizontal flow component in the L&dB aquitard that requires MLU to use a layered aquitard approach. Therefore, for this MLU example the aquitard was divided in four layers with thicknesses of 1/6, 1/3, 1/3 and 1/6 of the total aquitard thickness of 10 m. The aquitard's horizontal flow component and the storativity were simulated by three very thin "aquifer" layers located between the aquitard sublayers. The division in MLU of the upper aquifer in two layers of 1 and 9 m respectively, to simulate the dewatering at the top of the upper aquifer, is analogous to the L&dB multi-layer simulation.




**A layered aquitard consisting of 4 subaquitards and 3 horizontal flow layers.**

Three observation wells are located in the middle of the pumped layer and three in the middle of the overlying aquitard. In MLU these are aquifer layers no. 6 and 4 respectively. The pumping rate is 180 $m^3$/d and not 240 $m^3$/d as stated in the 1995 publication.

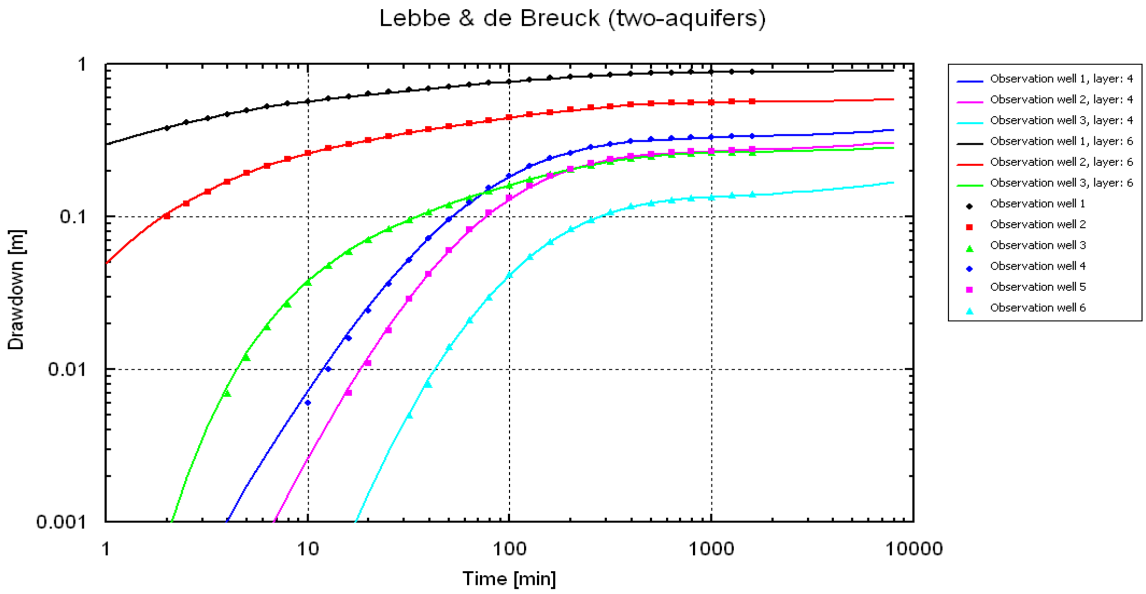
The fully populated MLU "Aquifer system" tab is shown below.




| Aquifer | Base [m] | Thickness [m] | K [m/d] | Code | T [m²/d], c [d] | # | Code | S [-] | # | Name |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -1 | 1 | 10 | T1 | 10 | | S1 | 0.2 | | water table layer |
| | -1 | 0 | 9.090909 | c2 | 0.55 | | S'2 | 0 | | resistance layer |
| 2 | -10 | 9 | 10 | T2 | 90 | | S2 | 0.004 | | upper aquifer |
| | -11.66 | 1.66 | 0.097647 | c3 | 17 | | S'3 | 0 | | aquitard |
| 3 | -11.67 | 0.01 | 167 | T3 | 1.67 | | S3 | 0.00133 | | aquitard storativity + hor. flow |
| | -14.99 | 3.32 | 0.100606 | c4 | 33 | | S'4 | 0 | | aquitard |
| 4 | -15.01 | 0.02 | 83 | T4 | 1.66 | | S4 | 0.00134 | | aquitard storativity + hor. flow |
| | -18.33 | 3.32 | 0.100606 | c5 | 33 | | S'5 | 0 | | aquitard |
| 5 | -18.34 | 0.01 | 167 | T5 | 1.67 | | S5 | 0.00133 | | aquitard storativity + hor. flow |
| | -20 | 1.66 | 0.097647 | c6 | 17 | | S'6 | 0 | | aquitard |
| 6 | -30 | 10 | 10 | T6 | 100 | | S6 | 0.0008 | | pumped aquifer |

**MLU "Aquifer system" tab – Two-aquifers (six-layer model).**

The MLU and L&dB results are shown in the "Time graphs" tab below.




**Lebbe & de Breuck drawdown results (dots) compared to the six-layer MLU results (curves).**

The differences between the computed drawdowns of MLU and the L&dB numerical model are typically 0.001 m or less. Only the early values of the two observation wells closest to the pumping well show significant differences in the pumped aquifer. For example, the drawdown in the observation well at 5.01 m is computed by L&dB to be 0.286 m, while MLU yields 0.298 m. To sort out what causes this slight difference, an accurate finite element (MicroFEM) model was used as an independent check. A fine grid was constructed, especially around the pumping well in the center of the model. The average nodal spacing between the center and the observation well at 5.01 m is less than 0.2 m. With 100 time steps in the first stress period of 1 minute, drawdowns were computed to be 0.2952, 0.2958 and 0.2967 m when the time weighting factor for the implicit finite difference scheme was lowered from 1.0 to 0.8 and 0.5. The MLU results were tested for different Stehfest parameters. For n = 6, 10 and 16, results of 0.29844, 0.29826 and 0.29827 m were obtained. So, the results of MicroFEM and MLU are very close, 0.297 and 0.298 m respectively, while the finite difference model of L&dB calculates 0.286 m. Therefore, it seems justified to conclude that the differences between the L&dB and MLU models are largely due to inaccuracies of the finite difference solution technique.

L&dB used their synthetic example as an inverse model to estimate all hydraulic parameters. As input data for their inverse model they used all 151 simulated drawdowns from the six observation wells that were greater than 0.005 m. The total number of unknown parameters is nine (see next figure). L&dB choose a calibration strategy of first identifying a limited number of four parameters, and then gradually increasing this number until all nine parameters were included in the calibration process. In this way the total number of iterations and the associated computation time is kept under control. Although the MLU computations are much faster, the same policy of first trying to estimate the values of most sensitive parameters first and then gradually increasing this number by including less sensitive parameters in the optimization, is a useful approach in most cases. Starting with initial values that are quite different from the actual values, L&dB succeed in retrieving the properties of the pumped aquifer, the aquitard and the specific yield.

In MLU a similar parameterization process to find all nine parameters was attempted. A difference with the L&dB exercise is that in MLU the “observed drawdowns” are the computed results of L&dB’s finite-difference model, which vary somewhat from the analytical results. Starting with the same initial values as L&dB used (see figure below), all nine unknowns were estimated in a single run (see table below).

| Aquifer | Base [m] | Thickness [m] | K [m/d] | Code | T [m²/d], c [d] | # | Code | S [-] | # | Name |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -1 | 1 | 5 | T1 | 5 | 1 | S1 | 0.2 | 6 | water table layer |
| | -1 | 0 | 5 | c2 | 1 | 4 | S'2 | 0 | | resistance layer |
| 2 | -10 | 9 | 5 | T2 | 45 | 1 | S2 | 0.004 | 7 | upper aquifer |
| | -11.66 | 1.66 | 0.195294 | c3 | 8.5 | 5 | S'3 | 0 | | aquitard |
| 3 | -11.67 | 0.01 | 67 | T3 | 0.67 | 2 | S3 | 0.00133 | 8 | aquitard storativity + hor. flow |
| | -14.99 | 3.32 | 0.201212 | c4 | 16.5 | 5 | S'4 | 0 | | aquitard |
| 4 | -15.01 | 0.02 | 33 | T4 | 0.66 | 2 | S4 | 0.00134 | 8 | aquitard storativity + hor. flow |
| | -18.33 | 3.32 | 0.201212 | c5 | 16.5 | 5 | S'5 | 0 | | aquitard |
| 5 | -18.34 | 0.01 | 67 | T5 | 0.67 | 2 | S5 | 0.00133 | 8 | aquitard storativity + hor. flow |
| | -20 | 1.66 | 0.195294 | c6 | 8.5 | 5 | S'6 | 0 | | aquitard |
| 6 | -30 | 10 | 3 | T6 | 30 | 3 | S6 | 0.0008 | 9 | pumped aquifer |

Time units: Days Length units: Meters

**Lebbe & de Breuck six-layered aquifer system with initial values of the nine optimized parameter groups.**

Some of the hydraulic properties are grouped to a single parameter. Because the homogeneous aquitard is split into four aquitard layers with different thicknesses, the initial value of the vertical resistance of each of these layers is chosen proportional to their thickness. These four resistances are estimated as a single grouped parameter, indicated by the code “5”.

Similarly the transmissivities and storativities of the three layers with horizontal flow within the aquitard, are indicated by the codes “2” and “8”. Likewise, the 1 m water-table layer and the 9 m deeper part of the upper aquifer have the same horizontal conductivity. Their transmissivity ratio is 1: 9 and both hydraulic properties can be estimated as a single parameter, as shown by code “1” in the aquifer system table.

When the nine parameter groups of the aquifer system are estimated with MLU, five parameter groups can be accurately estimated: (T3, T4, T5), T6, (c3, c4, c5, c6), (S3, S4, S5) and S6. The other parameters can only be estimated within a rather large range (c2, S1), or they cannot be obtained at all (T1, T2, S2). However, these results (not presented here) are based on fitting the MLU computed drawdowns to the L&dB results, although both methods produce different results especially for the early-time drawdown data close to the pumping well.

To validate the inverse modeling technique of MLU in the same way as in the L&dB publication, all “observed” drawdowns were replaced by the MLU-computed drawdowns. With this adjusted input, rounded off to the nearest millimeter, all but one parameter (S2) were recovered.

**MLU estimated parameters when “observed drawdowns” were replaced by MLU-computed drawdowns.**

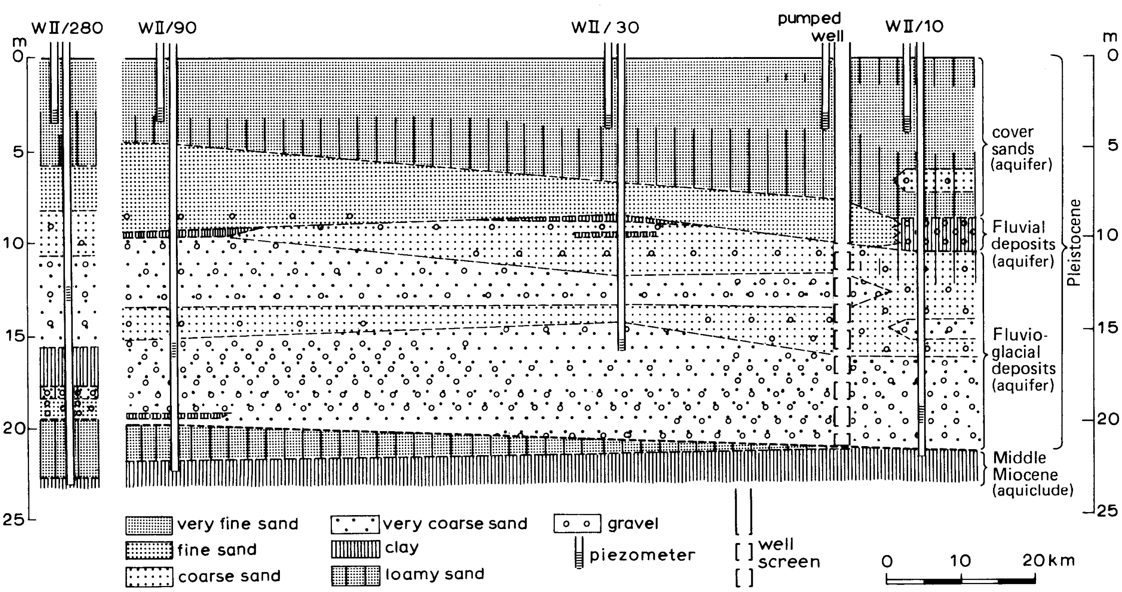
| Parameter | value | Standard | deviation | Actual value |
|---|---|---|---|---|
| T 1 | 10.2 | 2.4 | (23 % ) | 10 |
| T 2 | 91.5 | 21.4 | (23 % ) | 90 |
| T 3 | 1.68 | 5.277E-03 | ( 0 % ) | 1.67 |
| T 4 | 1.65 | 5.198E-03 | ( 0 % ) | 1.67 |
| T 5 | 1.68 | 5.277E-03 | ( 0 % ) | 1.67 |
| T 6 | 100.0 | 1.805E-02 | ( 0 % ) | 100 |
| c 2 | 0.44 | 0.11 | ( 24 % ) | 0.55 |
| c 3 | 17 | 0.014 | ( 0 % ) | 17 |
| c 4 | 33 | 0.028 | ( 0 % ) | 33 |
| c 5 | 33 | 0.028 | ( 0 % ) | 33 |
| c 6 | 17 | 0.014 | ( 0 % ) | 17 |
| S 1 | 0.1945 | 0.0174 | ( 9 % ) | 0.2 |
| S 2 | 0.00010 | 0.01548 | (very large) | 0.004 |
| S 3 | 0.00132 | 1.230E-06 | ( 0 % ) | 0.00133 |
| S 4 | 0.00135 | 1.268E-06 | ( 0 % ) | 0.00134 |
| S 5 | 0.00132 | 1.230E-06 | ( 0 % ) | 0.00133 |
| S 6 | 0.00080 | 4.822E-07 | ( 0 % ) | 0.0008 |

Using the same initial parameter values of L&dB as initial MLU input, the optimization process requires 22 iterations to reduce the initial sum of squares of 45.8206 $m^2$ to practically zero. The whole process takes about 10 seconds on a 1.8 GHz Dual CPU PC.

## Vennebulten

| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Phreatic | 3 | 1 | 2 | 4 | T3 c3 S1 S3 |

Although the previous two examples (Moench and Two-aquifers) discuss phreatic aquifer systems, these examples are based on synthetic data. The analysis of real aquifer tests in phreatic systems can be more troublesome as the present example will demonstrate. We will now look at the Vennebulten aquifer test, which K&dR (p. 104-106) used as an example to illustrate the curve-fitting method for a delayed water-table response. They conceptualized the complex stratigraphy shown below as a single phreatic aquifer




**Lithological cross-section of the pumping-test site 'Vennebulten' (from K&dR, 2000)**

At the Vennebulten site an 11 m thick aquifer is overlain by 10 m of fine sediments. The aquifer was pumped for 25 hours at a constant rate of 873 $m^3$/day. Piezometers were placed in both the pumped aquifer and near the water table at four distances from the well. Only the drawdown data from the shallow and deep piezometer at 90 m are presented by K&dR (Table 5.1).

In their graphical analysis K&dR only used the deep piezometer drawdown data to estimate values of the aquifer's T (KD), S, vertical conductivity, and specific yield. The easiest way to set up MLU for this case is as a two-layer hydrogeologic model with a single aquifer and a water-table layer on top separated by a resistance layer.

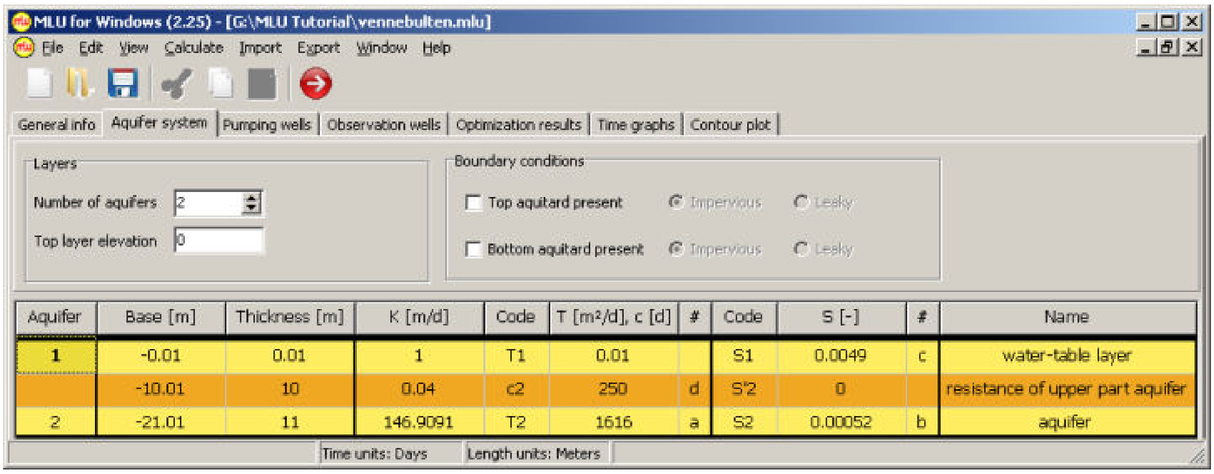
To follow the two-layer conceptual model discussion below, the user must open and modify the vennebulten.mlu project. The modification steps include:

- Convert to a two-layer model

- Input the K&dR results as initial values (T2 = 1616 $m^2$/d, c2 ≈ 250 d ($k_v$ = 0.04 m/d), S1 = 4.9x$10^{-3}$ and S2 = 5.2x$10^{-4}$)
- Move the pumping well to layer 2
- Move the deep piezometer to layer 2
- Un-check the shallow piezometer
- Set # codes to optimize T2, S2, S1, and c2

TIP> If you save the modified two-layer Vennebulten project, use a different name because the vennebulten.mlu file will be used later in this example.

The fully populated un-optimized two-layer model is shown below.

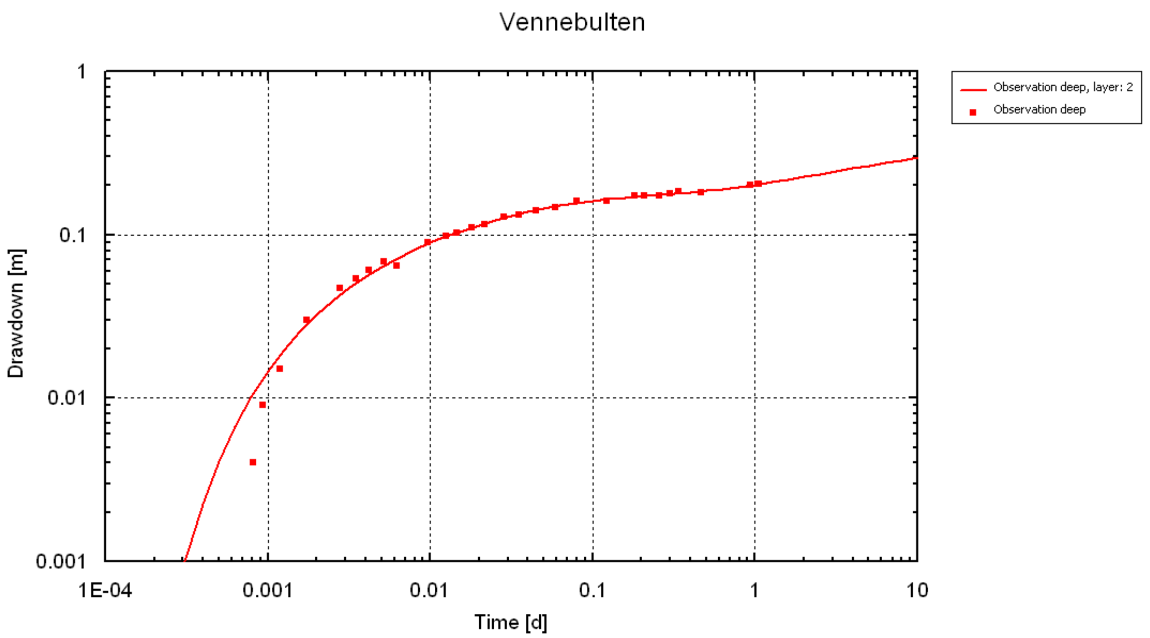



**Modified two-layer Vennebulten phreatic model).**

Using linear drawdown curve fitting, optimization of the four relevant parameters reduces the sum of squares from 0.0008 to 0.0003 $m^2$ and produces the following results.

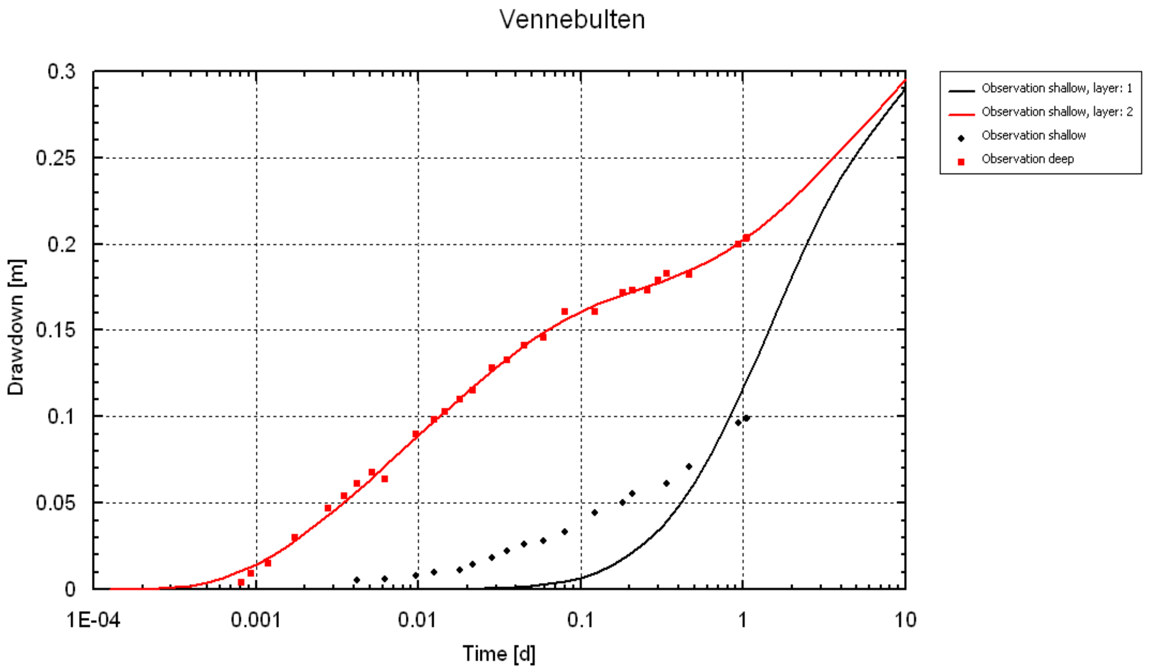
| | | | | | |
|---|---|---|---|---|---|
| T1 | 0.01 | | S1 | 0.0050 | 18% |
| c2 | 182 | 16% | S'2 | 0 | |
| T2 | 1568 | 4% | S2 | 0.00058 | 4% |

T2, S1 and S2 are independent of the assumed layer thicknesses and their values are very close to those of K&dR. The forth parameter, c2 = 182 days, is harder to compare with the Kv = 0.04 m/d result of K&dR. A resistance of 182 days as obtained by MLU for the 10 m thick fine grained, unpumped upper part of the aquifer equates to an average Kv of 0.055 m/d. The layered MLU model agrees better with the lithological description and cross section, while K&dR adopt a less realistic model based on a homogeneous aquifer and a fully penetrating well.




**Vennebulten test: Comparison of measured drawdowns and MLU computed drawdowns in the deep piezometer generated using the two-layer model with linear drawdown curve fitting.**

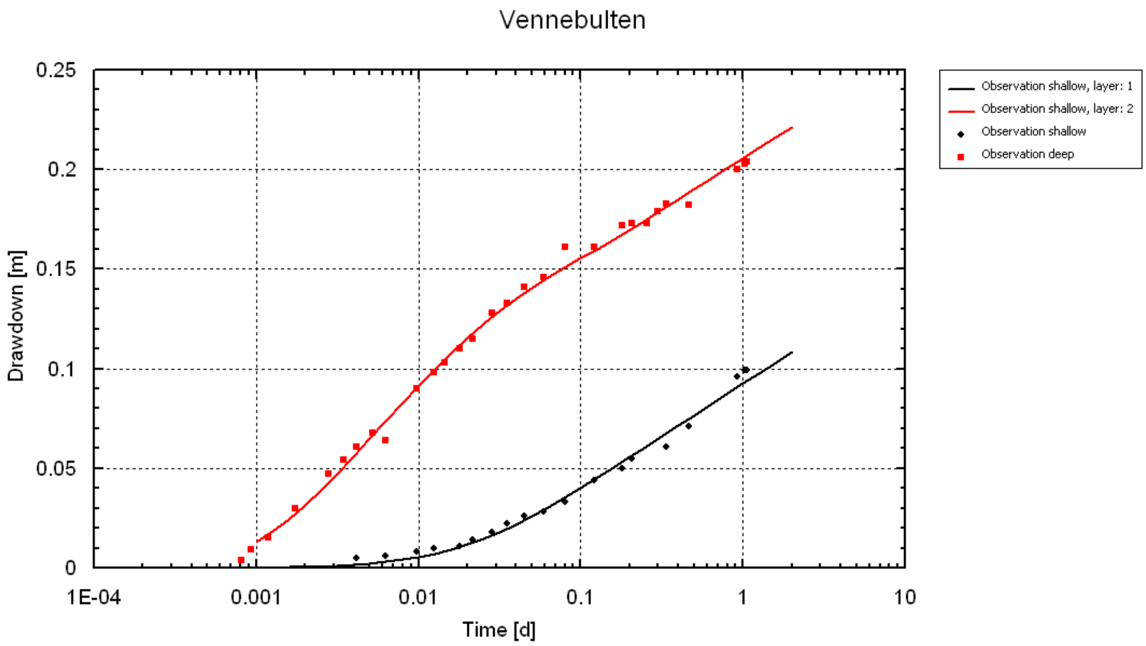
If we now add the shallow piezometer data to the graph (add the piezometer by checking it in the "Observation well" tab and moving it to layer 1 without optimizing) the MLU calculated water-table drawdown doesn't match the observed drawdowns.




**Incompatibility of observed shallow drawdown and computed water-table response of the Vennebulten test (two-layer model).**

The model needs further optimization. A renewed optimization with both piezometers seems the most appropriate. When the transmissivity of the top layer is added to the parameters to be estimated, a good fit can be obtained with the following set of five optimized parameters.

| | | | | | |
|---|---|---|---|---|---|
| T1 | 1700 | 10% | S1 | 0.0012 | 29% |
| c2 | 61.5 | 18% | S'2 | 0 | |
| T2 | 1316 | 7% | S2 | 0.00059 | 4% |

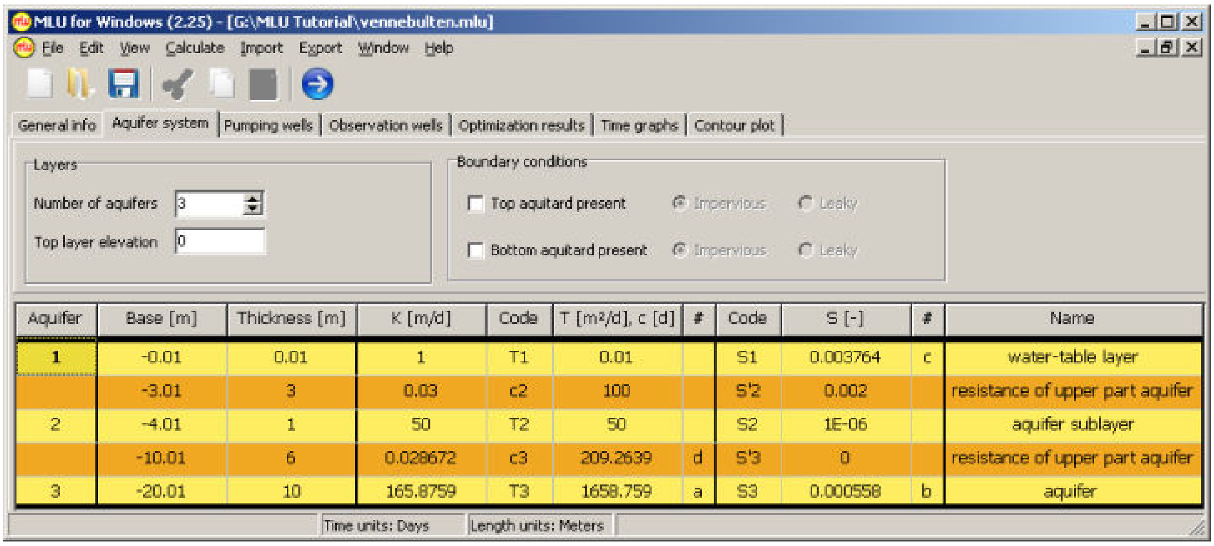



**A good fit between MLU computed drawdowns and field data is not necessarily the result of a valid model (two-layer model).**

Although the fit between the MLU results and field data is very good, the drilling data show no evidence of a highly transmissive shallow unit (very fine sand, see cross-section). This is a clear demonstration that a good fit is no proof that a valid conceptual model has been created. The risk of finding a “false fitting model” increases when the number of observation wells is small. In this case we only have data from two observation wells available, while the test was actually conducted with eight observation wells.

Now that just parameter estimation has been shown not to help us, we have to re-examine our conceptual model. Only the shallow piezometer data cause interpretation problems. Because its depth is about 3 m, it is probably located below the water table. If the shallow piezometer drawdown data do not represent the falling water table but rather the drawdown at a deeper level, the MLU model will require an additional aquifer sublayer with horizontal flow.

However, an additional aquifer layer implies four more parameters. Now that we know that a two-layer model exists, we may expect an infinite number of three-layer models. This means a choice must be made as to which parameters have to be kept fixed at some assumed value. With the few data we have from the test site, this becomes an intuitive task. Two such solutions are presented here, one with four and one with six estimated parameters (open the original vennebulten.mlu file to access the three-layer model and see figure below). In both conceptual models some horizontal flow in the fine grained upper part of the aquifer is assumed.




**MLU "Aquifer system" tab – Vennebulten (three-layer model).**

The MLU optimized results using linear drawdown curve fitting are summarized below:

**Three-layer Vennebulten model with four estimated parameters.**

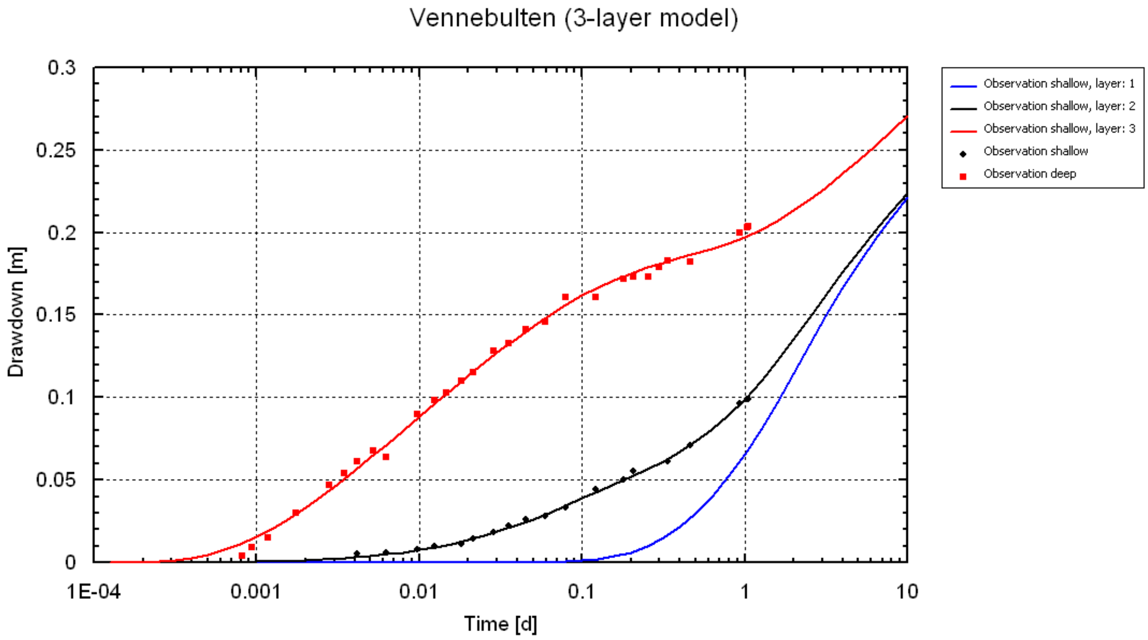
| | | | | | |
|---|---|---|---|---|---|
| T1 | 0.01 | | S1 | 0.0038 | 12% |
| c2 | 100 | | S'2 | 0.002 | |
| T2 | 50 | | S2 | 0.000001 | |
| c3 | 209 | 4% | S'3 | 0 | |
| T3 | 1659 | 1% | S3 | 0.00056 | 3% |

**Three-layer Vennebulten model with six estimated parameters.**

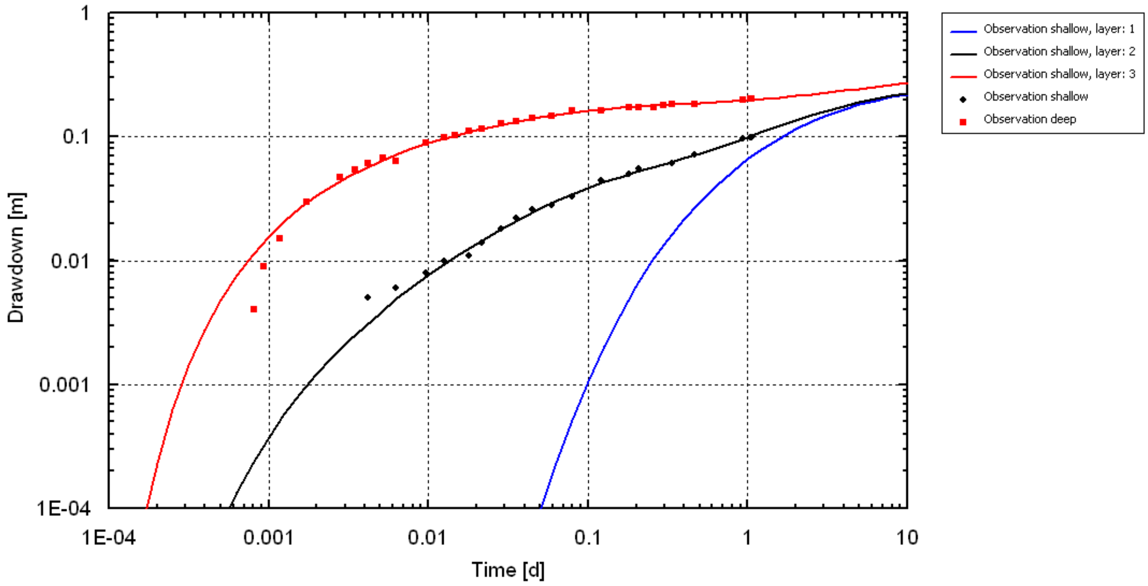
| | | | | | |
|---|---|---|---|---|---|
| T1 | 0.01 | | S1 | 0.0060 | 17% |
| c2 | 77 | 19% | S'2 | 0.0015 | 38% |
| T2 | 10 | | S2 | 0.0001 | |
| c3 | 203 | 16% | S'3 | 0 | |
| T3 | 1629 | 4% | S3 | 0.00057 | 5% |

Both parameter sets generate drawdown curves that fit the field data equally well, producing the same sum of squares of 0.0005 $m^2$. The transmissivity and storativity of the pumped aquifer are only slightly different, however both transmissivities are somewhat higher than the 1569 $m^2$/d estimated using the two-layer model.

TIP> There is no direct basis for choosing the fixed values of T2=50 and T2=10 $m^2$/d in the two solutions. We could have chosen 25 or 5 $m^2$/d, respectively. A similar reasoning applies to S2. The goal is to choose values that are realistic and that help produce a good match between the MLU estimated drawdowns and the field data.




**Vennebulten test: Comparison of the measured and MLU computed drawdowns in the shallow and deep piezometers generated using the three-layer model with four estimated parameters and a linear drawdown axis. The blue curve shows the computed water-table response.**




**Vennebulten test: Comparison of the measured and MLU computed drawdowns in the shallow and deep piezometers generated using the three-layer model with four estimated parameters and a log drawdown axis. The blue curve shows the computed water-table response.**

Similar graphs can be created for the three-layer model with six estimated parameters.

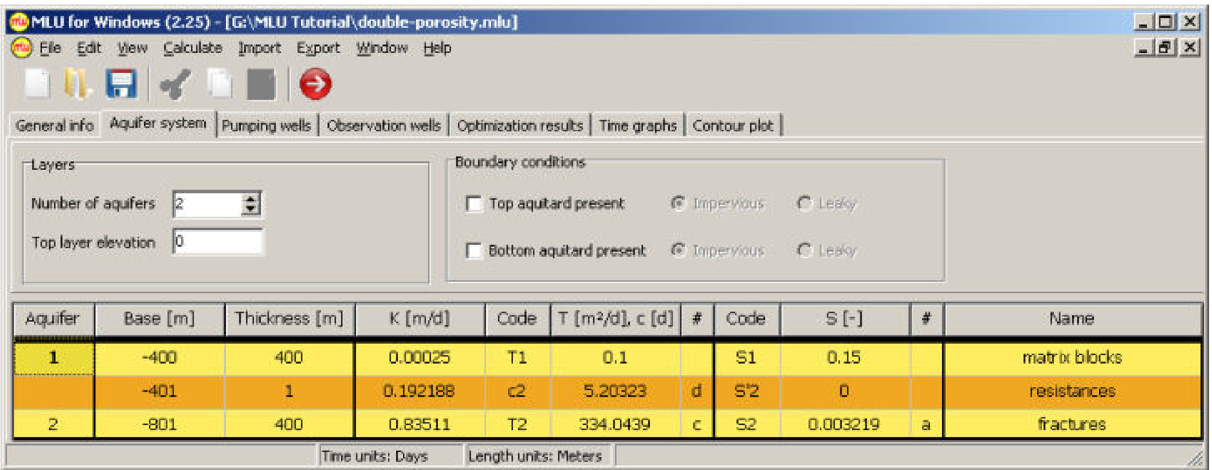
## Double porosity

| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Confined | 2 | 1 | 1 | 3 | T2 c2 S2 |

In a double-porosity type aquifer two co-existing media are recognized with their own characteristic properties: the fractures of high conductivity and low storage capacity, and the matrix blocks of low conductivity and high storage capacity. The radial flow towards the well in such systems is entirely through the fractures. The drawdown in the fractures causes interporosity flow from the matrix blocks into the fractures, also referred to as block-to-fissure flow. This flow can be assumed to be of a linear (pseudo-steady state) or a diffusive (transient) type. The generalized fracture skin model proposed by Moench (1984) is based on a combination of both. Linear flow implies that the rate of flow is proportional to the average difference in heads between fissures and blocks, while diffusive flow is based on the development of a head distribution in the matrix material of the blocks. A short description of the double-porosity concept can be found in the Chapter 17 Introduction of K&dR (2000).

Double-porosity with linear interporosity flow can be simulated using MLU by adding a very low-transmissivity aquifer to the top of the system representing the matrix blocks, separated from the fracture flow layer by a zero-storativity aquitard, similar to phreatic aquifers. When the block geometry of a double-porosity formation can be idealized as infinite slabs, solutions for diffusive block-to-fissure flow are identical to a confined aquifer-aquitard system with storage in the aquitard. The generalized model of Moench requires a zero-storativity aquitard to represent the fracture skin, and an aquitard with storage for the diffusive flow. See Hemker & Maas (1987) for a short discussion on the similarity between layered and fissured formations.

In this example we will work with the double-porosity test data published by Moench (1984). The same test is also used by K&dR (p. 258-261) to demonstrate a graphical analysis method based on a linear interporosity flow model and straight-line matching of the pumping well drawdowns.



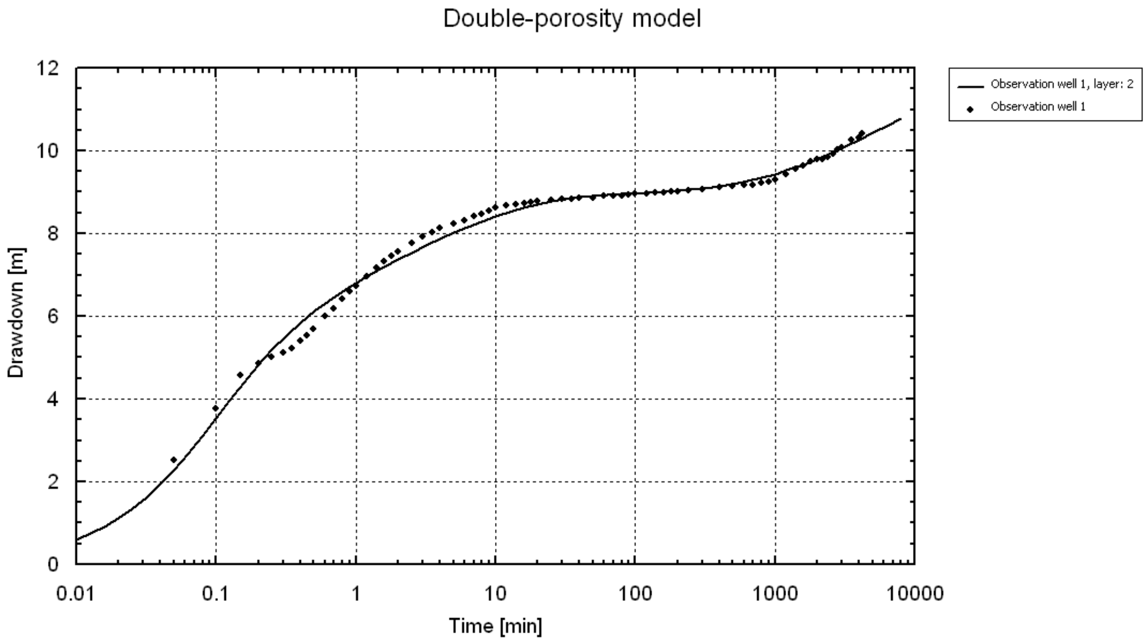

**MLU "Aquifer system" tab – Double-porosity (two-layer fractured aquifer).**

When all four parameters (S1, c2, T2, S2) are estimated in a linear interporosity MLU model, we find an almost perfect fit between the computed drawdown and the field data, but the calculated value for the matrix storativity is as high as 2.65. Also, all parameters are nearly 100% correlated: an obvious example of overparameterization.

To obtain a unique solution we have to set one of the parameters to a fixed value. When the matrix storativity is set to 0.15 (calculated by K&dR, p. 261) the other three parameters can be found within narrow limits as shown below.

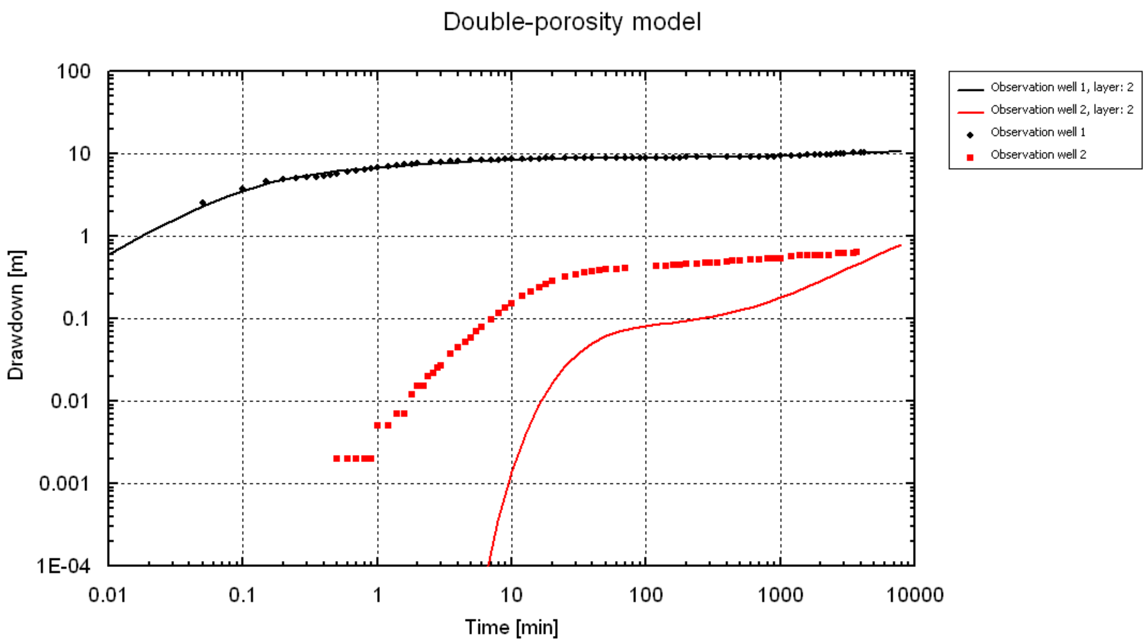
**Two-layer Double-porosity model with three estimated parameters.**

| | | | | | |
|---|---|---|---|---|---|
| T1 | 0.1 | | S1 | 0.15 | |
| c2 | 5.20351 | 11% | S'2 | 0 | |
| T2 | 334.045 | 1% | S2 | 0.003219 | 7% |




**Double-porosity model: Comparison of measured and MLU computed drawdowns in the pumped well generated using the two-layer linear interporosity model with three estimated parameters and linear drawdown curve fitting.**

This solution fits the drawdown data reasonably well. The transmissivity is equal to that of K&dR (2000), while the fracture storativity is twice as high. The publication by Moench (1984) also presents drawdowns measured in an observation well at 110 m from the pumping well. When we add these data to the graph, a log-drawdown plot shows that our linear interporosity model is not as good as initially thought. New optimization runs prove that further parameter estimation with the same linear model do not lead to satisfactory results for both curves. Therefore, a better conceptual model has to be developed.




**Double-porosity model: Comparison of measured and computed drawdowns in the pumped and observation well generated using the two-layer linear interporosity model with three estimated parameters and linear drawdown curve fitting.**

Moench (1984) developed a fracture skin model that fits the measurements much better. It is based on a resistance layer with negligible storage capacity between the matrix and the fractures, but diffusive flow within the matrix is also considered. Diffusive interporosity flow can be modeled in MLU by the release of water from aquitard storage. A fracture skin requires the resistance of another aquitard. In MLU these aquitards must be separated by an additional aquifer layer with negligible transmissivity.

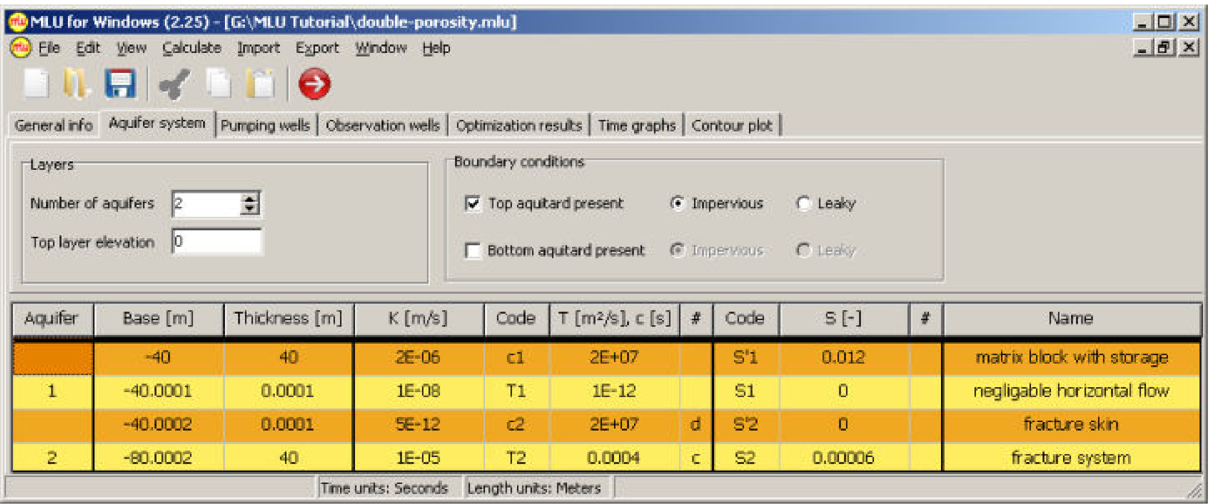
There are five slab-shaped blocks of 80 m thick each in the Moench model, separated by fractures. The total thickness of the double-porosity aquifer is 400 m.

Moench computes the following parameters:

- Fracture system horizontal conductivity and specific storage: $1 \times 10^{-5}$ m/s and $1.5 \times 10^{-6}$ $m^{-1}$
- Matrix system vertical conductivity and specific storage: $2 \times 10^{-6}$ m/s and $3 \times 10^{-4}$ $m^{-1}$
- Fracture skin resistance: $2 \times 10^{-7}$ s.

In MLU we can set up a complex layered system of five matrix layers (the slab-shaped blocks) bounded above and below by layers with horizontal flow (the fracture system).
In this case each matrix layer must consist of an aquitard with storage, bounded above and below by other aquitards to represent the resistance of the fracture skin. In this way three aquitards are required in MLU to model one matrix layer, which adds up to 15 aquitards all together.

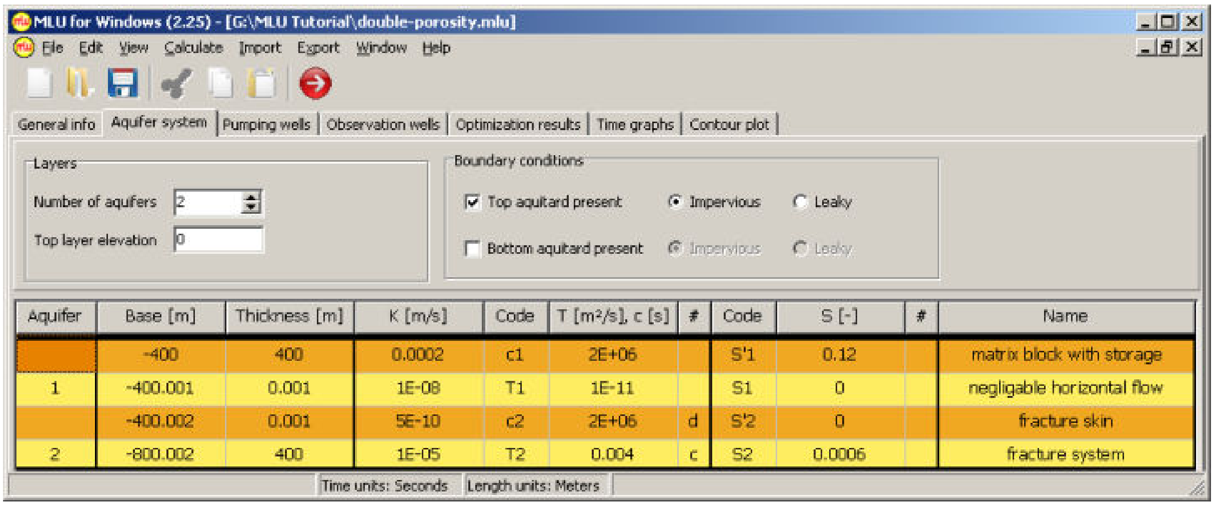
Because the flow in and around each matrix layer is the same at any time, while the flow out of the matrix layer (upward in the upper half and downward in the lower half) is also symmetrical, we can simplify the MLU model by considering only one tenth of the aquifer. We only need to consider the flow in a 40 m thick layer of the aquifer, between the middle of a fracture and the middle of a matrix layer. In this case both the pumping rate and the wellbore storage must also be reduced to 10%. The effect of the wellbore storage is proportional to the square of the casing radius, so the radius of 0.11 m must be reduced to 0.11 * sqrt(1/10) = 0.035 m. The resulting aquifer system table summarizing the conceptual model and using the hydraulic properties given by Moench (Table 4) is shown below.




| Aquifer | Base [m] | Thickness [m] | K [m/s] | Code | T [m²/s], c [s] | # | Code | S [-] | # | Name |
|---|---|---|---|---|---|---|---|---|---|---|
| | -40 | 40 | 2E-06 | c1 | 2E+07 | | S'1 | 0.012 | | matrix block with storage |
| 1 | -40.0001 | 0.0001 | 1E-08 | T1 | 1E-12 | | S1 | 0 | | negligable horizontal flow |
| | -40.0002 | 0.0001 | 5E-12 | c2 | 2E+07 | d | S'2 | 0 | | fracture skin |
| 2 | -80.0002 | 40 | 1E-05 | T2 | 0.0004 | c | S2 | 0.00006 | | fracture system |

**Double-porosity fracture skin model for 10% of the total aquifer thickness.**

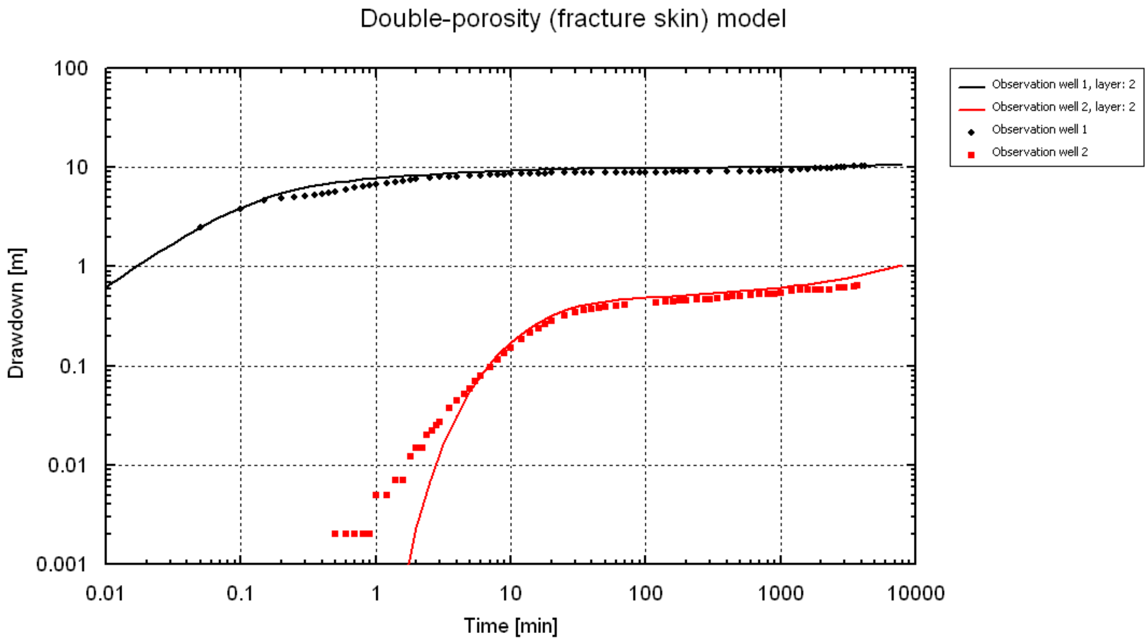
Instead of considering only 10% of the aquifer system we can incorporate the full aquifer system by multiplying the transmissivities, the storativities and the discharge rate by ten, and divide the resistances by ten. See Appendix B for an explanation. In this way the real discharge rate and casing radius can be used while all computed drawdowns remain unaltered.




| Aquifer | Base [m] | Thickness [m] | K [m/s] | Code | T [m²/s], c [s] | # | Code | S [-] | # | Name |
|---|---|---|---|---|---|---|---|---|---|---|
| | -400 | 400 | 0.0002 | c1 | 2E+06 | | S'1 | 0.12 | | matrix block with storage |
| 1 | -400.001 | 0.001 | 1E-08 | T1 | 1E-11 | | S1 | 0 | | negligable horizontal flow |
| | -400.002 | 0.001 | 5E-10 | c2 | 2E+06 | d | S'2 | 0 | | fracture skin |
| 2 | -800.002 | 400 | 1E-05 | T2 | 0.004 | c | S2 | 0.0006 | | fracture system |

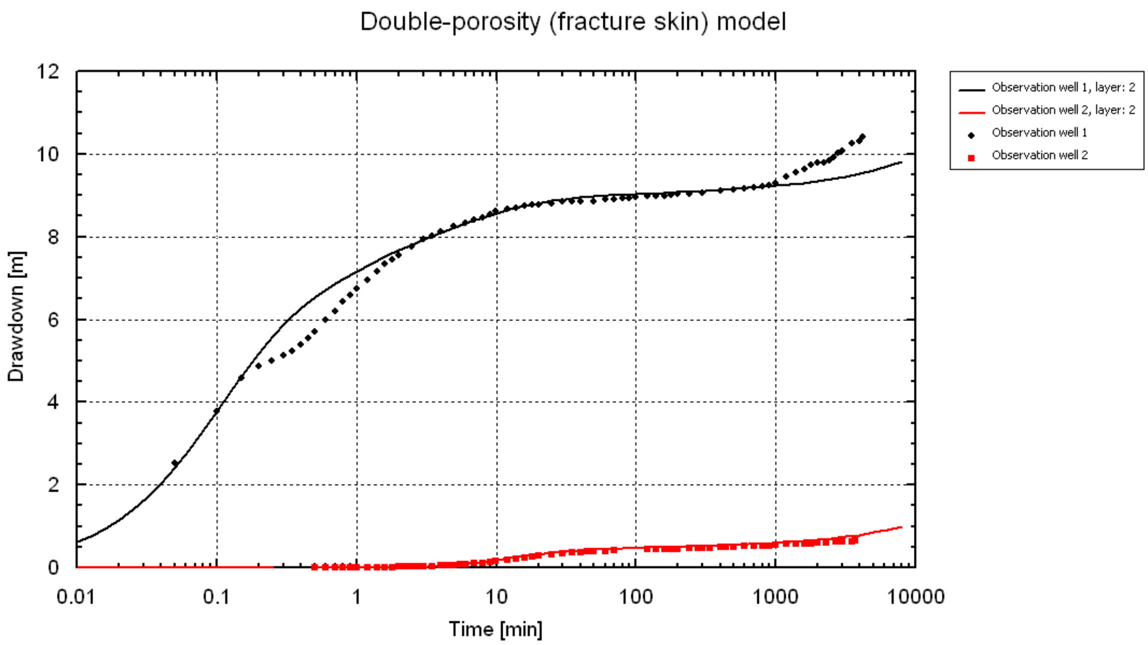
**Double-porosity fracture skin model for the total aquifer thickness.**

The computed drawdowns are slightly higher than measured, but the results compare very well with those of Moench (Figure 13).




**Double-porosity fracture skin model: Comparison of the measured drawdowns and MLU computed drawdowns in the pumped well and the observation well generated using the hydraulic properties given by Moench.**

With this revised conceptual model, further parameter estimation is possible. For example, when the fracture conductivity is estimated with linear drawdown curve fitting, the transmissivity increases from 345.6 to 378.2 $m^2$/d, which reduces the sum of squares from 46.9 to 10.3 $m^2$. The resulting deviation between the model and the measurements is best shown on a linear drawdown scale.




**Double-porosity fracture skin model with estimated fracture conductivity using linear drawdown curve fitting.**

The overall conclusion is that neither the linear model nor the fracture skin model fit the available measurements in both wells properly. The real hydrogeologic flow conditions likely deviate too much from those of the proposed conceptual models. An additional complication is that the two drawdown curve fitting methods (linear and logarithmic) have problems fitting both wells at the same time. Because of the large differences in measured drawdown data, linear drawdown curve fitting tends to ignore the small drawdown measurements in the observation well, while log drawdown curve fitting yields a solution where the pumping well data deviate substantially on a linear scale.

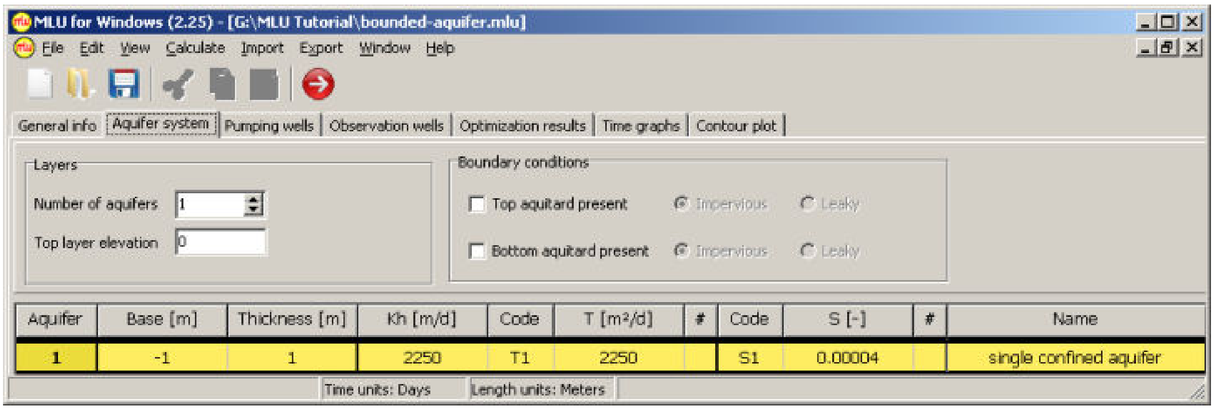
## Bounded aquifer

| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Confined | 1 | 187 | 13 | 0 | verification |

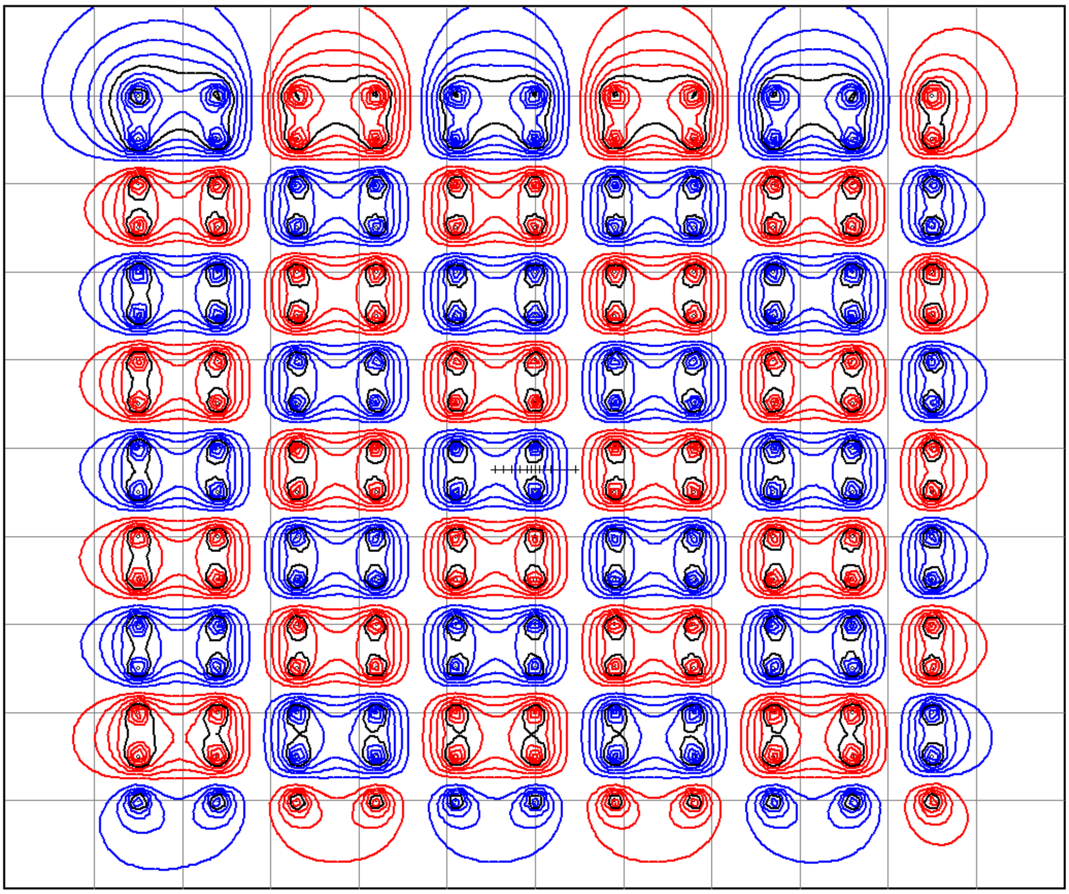
When an aquifer system has finite horizontal extent and the boundaries (up to 4) are straight and parallel or intersecting at right angles, an analytical solution based on image wells is possible. The boundaries must be present in all layers of the system and each boundary may be either a no-drawdown (recharging) boundary or a no-flow (barrier) boundary. Image well theory is explained in most text books. K&dR (Chapter 6) show diagrams of image well configurations for many types of boundary conditions.

K&dR give no examples, therefore for this example we will use one from Ségol's (1994) book "*Classic Groundwater Simulations*", page 49-51, Case 2.2, Alternative B. It is a synthetic well flow problem used originally to compare the results of analytical and numerical solutions. A single rectangular confined aquifer of 9000 by 5000 $m^2$ has a well at the center, no-flow boundaries along the south and west sides, and fixed head boundaries along the north and east sides.

For this case the number of image wells is theoretically infinite. The MLU solution uses (11 * 17) – 1 = 186 image wells. As many image wells are chosen North and South of the well, and as many East and West of the well. The configuration of discharge and recharge wells can be checked in the MLU "Contour plots" tab.

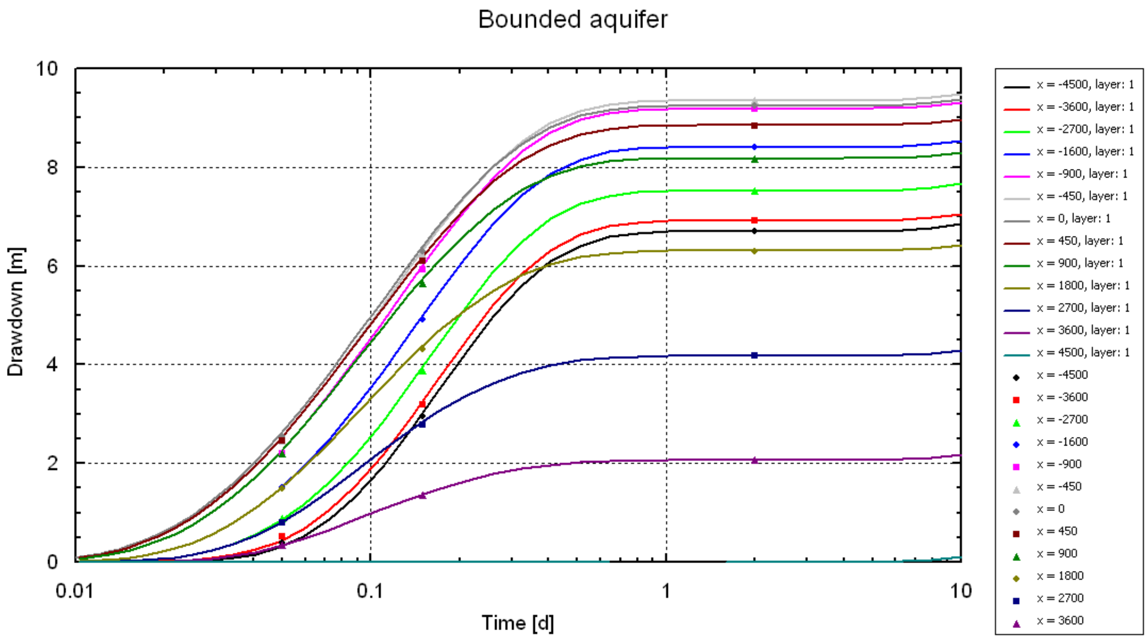



**MLU "Aquifer system" tab – Bounded-aquifer.**



**One pumping well and 186 image wells with their drawdown and build-up cones.**

Drawdowns are obtained along the southern boundary at 13 locations and for 3 times: 0.05, 0.15 and 2 days. The MLU results are compared with numerical results obtained with the finite element model MicroFEM as shown below.



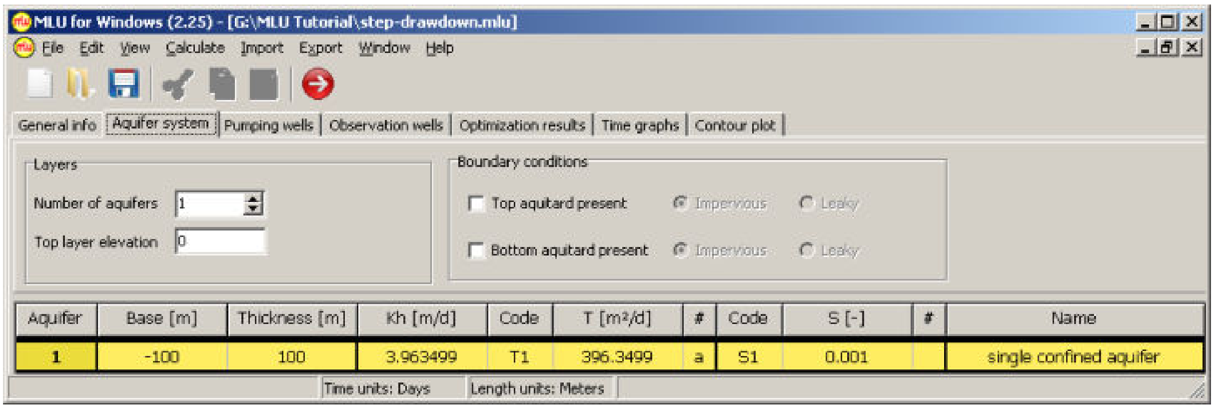

**Comparison of numerically computed drawdowns (dots) and MLU computed drawdowns (curves) along the southern no-flow boundary.**

Because of the recharge boundaries a steady state will be reached. The image well solution shows that this takes one or two days. However, after five days all curves show that drawdown starts increasing again. This increase is due to the finite number of image wells.

## Step-drawdown

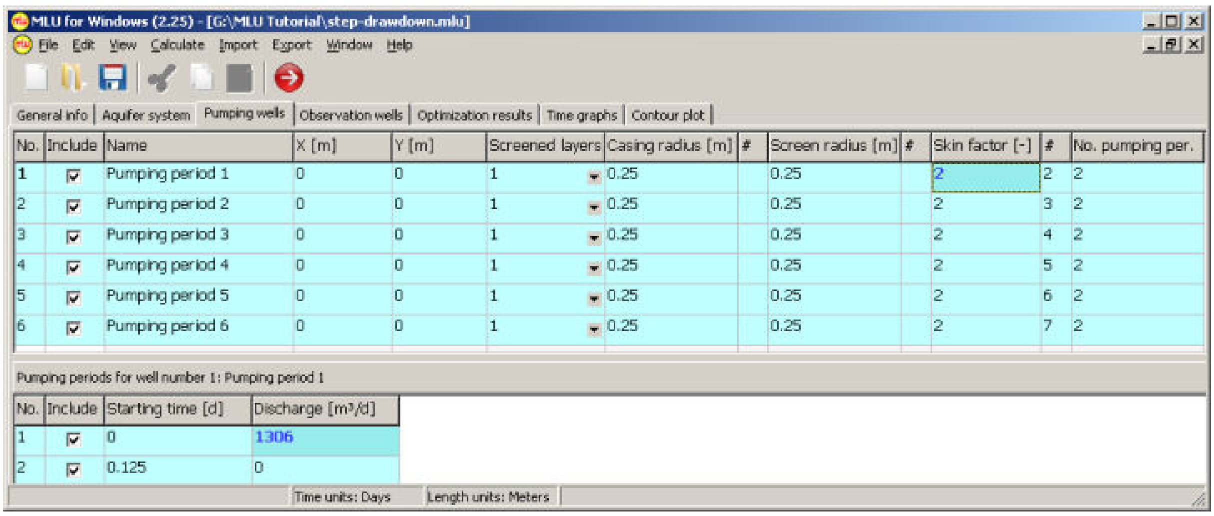
| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Confined | 2 | 6 | 6 | 7 | T1 sk1 up to sk6 |

To illustrate how MLU can be used to analyse a step-drawdown test we will look at the classical example of Clark (1977, Table 1), also used by K&dR (2000, p. 203-205). Drawdown series are given for six discharge rates, while the residual drawdown data during recovery are presented in K&dR Table 15.3 (p. 235). Additional information, such as well and casing radii, or an estimate of storativity is missing.

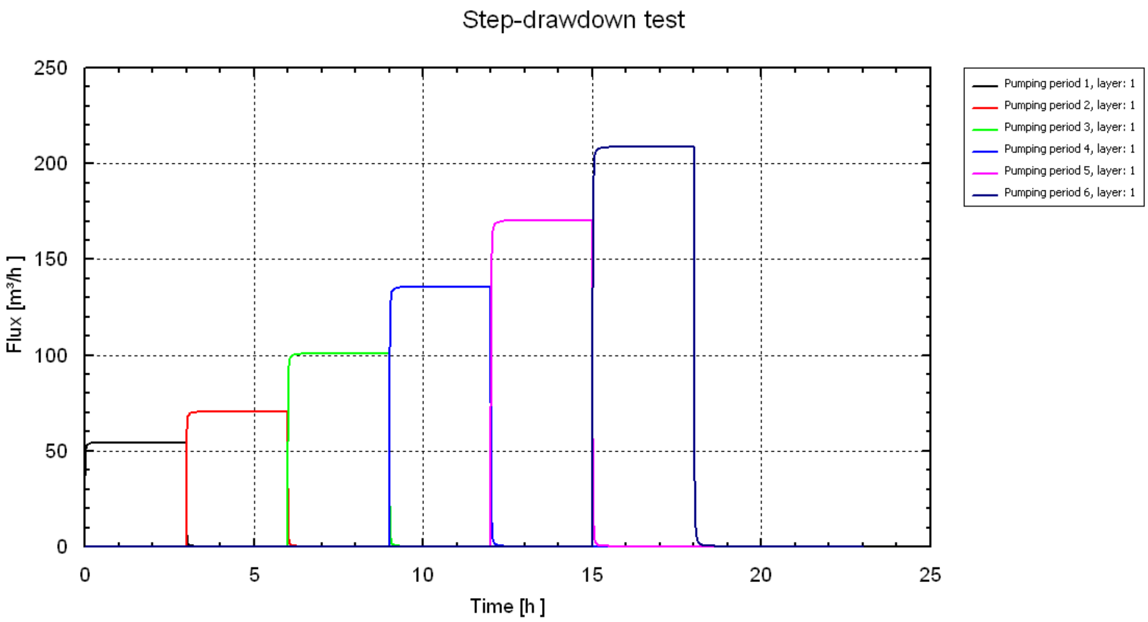



**MLU "Aquifer system" tab – Step-drawdown (confined aquifer)**

In MLU each pumping well can only have one value for the skin factor. Because one of the purposes of a step-drawdown test is to find out how the skin factor increases with the discharge rate, a new pumping well has to be defined for each pumping period. All of these pumping wells are located at the same location and they pump in succession with increasing pumping rates, each for a period of 180 minutes (3 hours).




**MLU "Pumping wells" tab – Step-drawdown, before estimating the transmissivity and skin factors.**

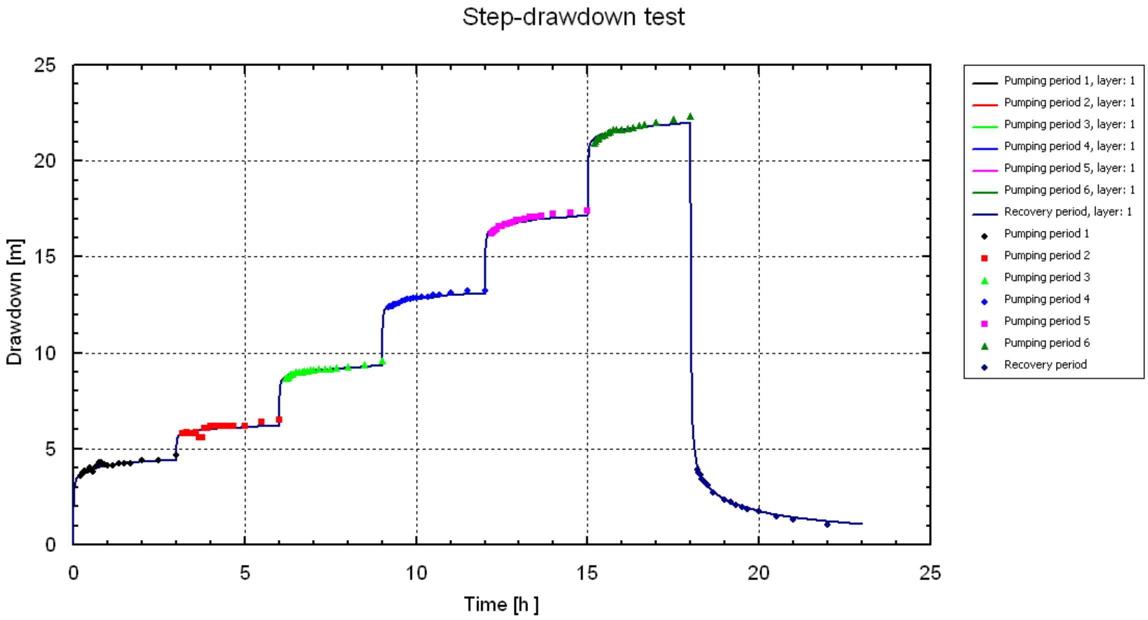



**Stepwise increasing flow rate during step-drawdown test.**

When the storativity is set to 0.001 and the pumping well casing and screen radii are set to 0.25 m, a good fit can be obtained, especially when the data for the first 10 minutes of each pumping and recovery period are not included in the optimization process (un-check the data in the “Observation wells” tab).

| | | | | | |
|---|---|---|---|---|---|
| Results: | T1 = | 396 $m^2/d$ | ( 1% ) | | |
| | Sk 1 = | 1.28 | ( 7% ) | Q = | 1306 $m^3/d$ |
| | Sk 2 = | 1.69 | ( 5% ) | | 1693 |
| | Sk 3 = | 2.07 | ( 5% ) | | 2423 |
| | Sk 4 = | 2.42 | ( 4% ) | | 3261 |
| | Sk 5 = | 2.77 | ( 4% ) | | 4094 |
| | Sk 6 = | 3.19 | ( 3% ) | | 5019 |

The absolute values for the skin factors are dependent on the assumed storativity and on the pumping well casing and screen radii, but the difference between the highest and the lowest value remains close to 1.9. If we assume that the skin factor increases by 1.9 (from 1.28 to 3.19) when the discharge rate increases by 3700 $m^3/d$ (from 1306 to 5019 $m^3/d$), then the non-linear well loss coefficient (C-value defined in Eq. 14.1 of K&dR) can be computed:
$C = (1.9/3700) / (2 \pi T) \approx 2x10^{-7}\ d^2/m^5$.




**Stepwise increasing drawdown in the pumping well followed by a recovery period.**

When only the drawdown during the pumping periods is analyzed, a significantly smaller transmissivity of 324 $m^2/d$ is estimated (396 $m^2/d$ for pumping + recovery), and skin factors vary from almost zero to 1.35.

K&dR estimated lower transmissivities than MLU, but they noted a similar difference between the values obtained with and without the recovery data (352 and 265 $m^2/d$, respectively).

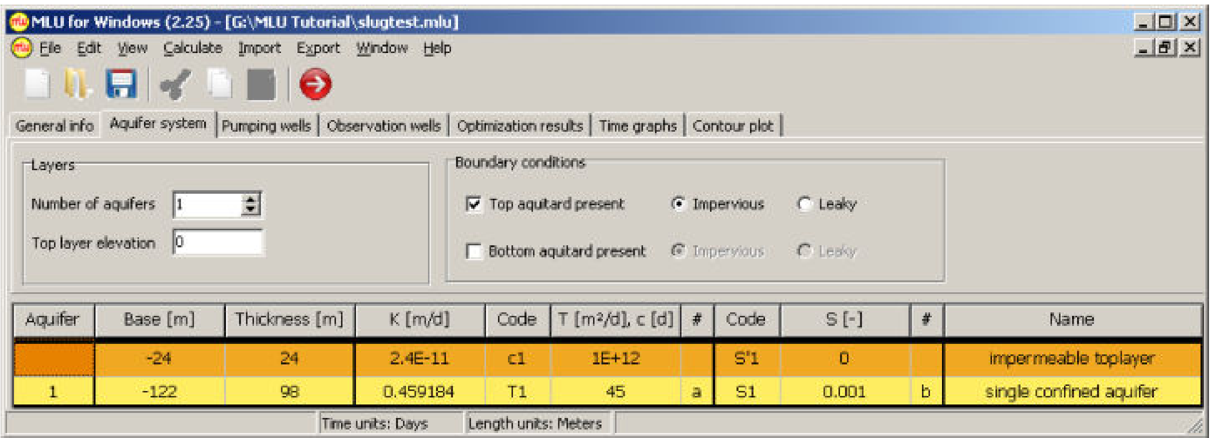
## Slugtest

| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Confined | 1 | 1 | 1 | 2 | T1 S1 |

During a slug test the recovery of the water level in the well is measured after a sudden removal (or addition) of a slug of water. This removal is not actually instantaneous but will take a small period of time. Because MLU can handle pumping periods as small as 0.1 second, all we have to do is convert the given volume of water and the 0.1 sec period of time into a corresponding pumping rate. Of course, a longer period, say 0.2, 0.5 or 1 second could be used to compute a discharge rate that is lower. However, the longer the "pumping" period the larger will be the difference between the time pumping started (t = 0, which is the MLU initial condition) and the time since the slug was fully inserted (t = 0 + Δt) the assumed time that recovery begins. When a period of only 0.1 second is used, the time difference is probably negligible for all measurements, otherwise Δt should be added to each of the recovery time measurements.

TIP> The surging action of rapidly inserting or withdrawing the slug, the physical limits of how fast a slug can be inserted or withdrawn, and the water dripping off a withdrawn slug may mask the Δt problem and in general degrade the accuracy of the earliest time data.

Although K&dR (2000) describe procedures to analyze slug tests, they give no examples. Therefore we will use the classical Dawsonville slug test example of Cooper et al. 1967, also used by Batu (1998) p. 667-668.



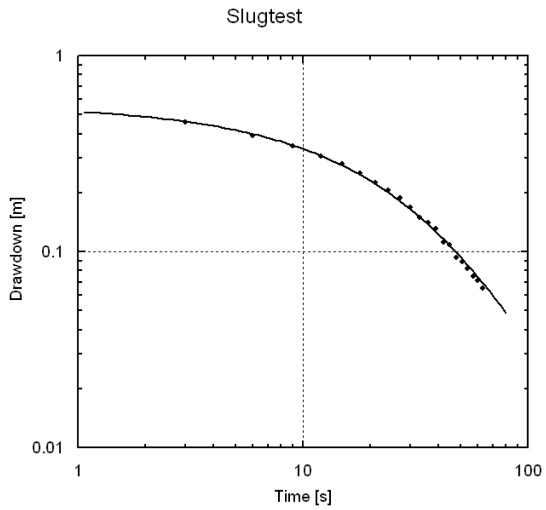

**MLU "Aquifer system" tab – Slugtest.**

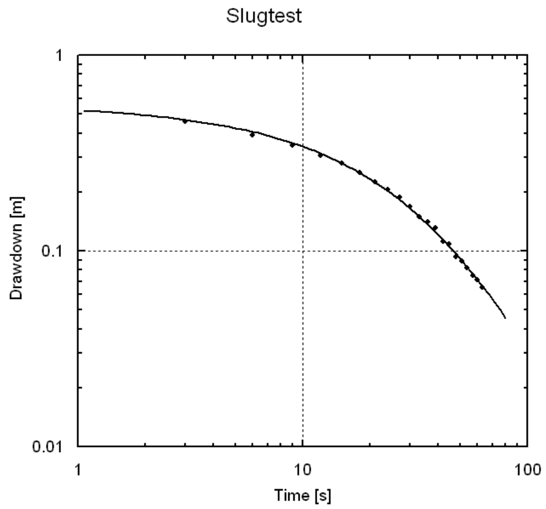
The casing diameter is 0.152 m. The slug of 10.16 liter is converted to a pumping rate of 0.1016 $m^3$/s during 0.1 sec. With Cooper's type curve method the transmissivity and storativity are estimated to be T = 45 $m^2$/d and S = $10^{-3}$.

Using the Cooper results as initial estimates, the MLU results are:

- Linear drawdown curve fitting: T = 40.5 (4%) and S = 0.0019 (29%)
- Log drawdown curve fitting: T = 47.2 (4%) and S = 0.00057 (41%).

The results are very sensitive to how the measurements are judged: whether the early or the later observations are considered more accurate.



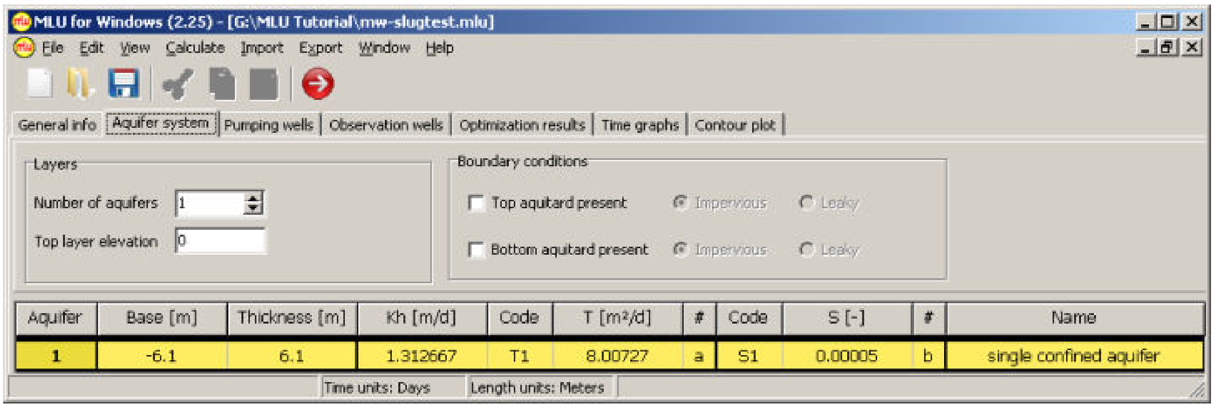

**Slug test data with linear drawdown curve fitting (left) and log drawdown curve fitting (right).**

## Mw-slugtest

| System | Model layers | Pumping wells | Obs. wells | Optimized parameters | |
|---|---|---|---|---|---|
| Confined | 1 | 1 | 2 | 3 | T1 S1 sk1 |

Simple slug tests are discussed in many textbooks. More models, methods and field examples of slug tests are discussed in the book "*The design, performance, and analysis of slug tests*" by J. J. Butler, Jr. (1998). For this MLU example we will use a multi-well slug test (pulse interference test) conducted at a site in Lincoln County, Kansas by Butler & Liu (B&L) published in 1997. The test well and the nearby observation well are screened in the same formation.

Test and observation well radii are known, but the observation well casing radius is set to zero as wellbore storage is assumed negligible because of the packer and transducer arrangement reported by B&L.



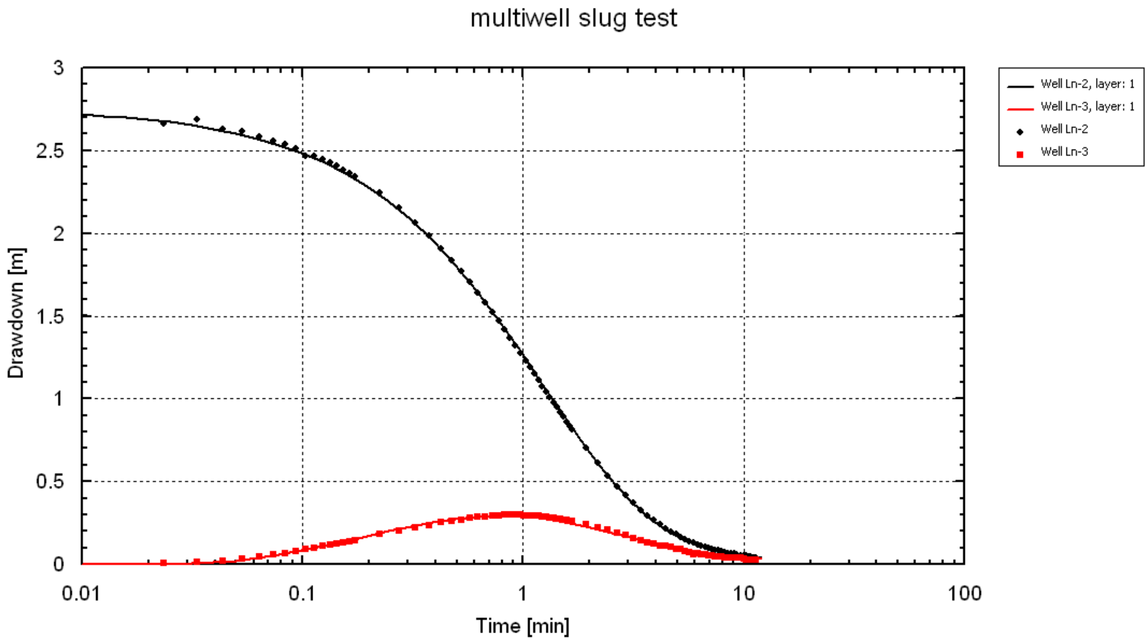

**MLU "Aquifer system" tab – Mw-slugtest.**

B&L's curve matching on a linear head scale yields: T = 6.5 $m^2$/d, S = 5.2x$10^{-5}$ and no skin, while an alternative curve matching procedure on a logarithmic head scale yields: T = 9.6 $m^2$/d, S = 4.0x$10^{-5}$ and a skin factor of 2.42.

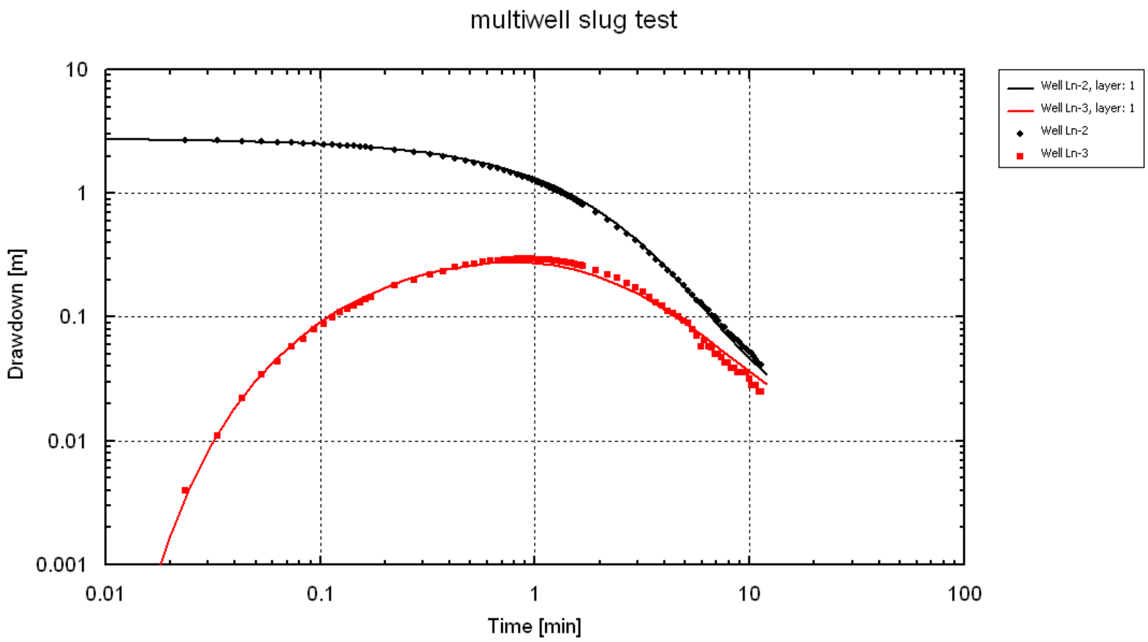
MLU results are intermediate:

- Linear drawdown curve fitting: T = 8.0 $m^2$/d (1 %), S = 5.0x$10^{-5}$ (1 %) and Skin = 0.65 (6 %).
- Log drawdown curve fitting: T = 9.1 $m^2$/d (1 %), S = 4.4x$10^{-5}$ (1 %) and Skin = 1.66 (6 %).

Note: The values in parentheses are the standard deviations expressed as a percentage of the estimated value (see "Optimization results" tab).




**Multi-well slug test: Comparison of measured drawdowns and MLU computed drawdowns generated with linear drawdown curve fitting.**




**Multi-well slug test: Comparison of measured drawdowns and MLU computed drawdowns generated with log drawdown curve fitting.**

# Appendix A

**MLU compared to other aquifer test software**

Because MLU is based on both analytical and numerical techniques, one may wonder how MLU compares to other software that is used for drawdown calculations, the analysis of aquifer tests and the design of well fields in layered systems.

Compared to the available aquifer test software, which are all based on a list of analytical solutions for particular flow conditions, MLU has the following unique combination of features:

- Multi-layer aquifer system
- Aquitard storage
- Multiple pumping and observation wells
- Multi-aquifer pumping wells
- Wellbore storage and skin
- Variable discharge rates
- Delayed drawdown response in observation wells.

MLU is based on a single mathematical solution and uses the same interface for all tests, such as: pumping tests, well tests, recovery tests, step-drawdown tests, slug tests, etc.

The large number of hydraulic properties of a multi-layer aquifer system excludes all classical methods based on graphical curve-fitting or searching for a best-fit straight line. The applied non-linear regression technique allows for a large number of parameter groups to be estimated at the same time based on a least-squares method. A comparison of the computed drawdown and field data can be checked by inspection of the residuals and graphically by a plot of the measured data and modeled results. Statistical information helps to find the best set of parameters to be estimated, and produces information about the accuracy of the results.

Compared to available 3D finite element and finite difference models, the advantages of MLU are:

- No finite element or finite difference grid is required
- No time steps are required
- Variable fluxes are accounted for into multi-aquifer well screens
- Drawdown within a pumping well accounts for well and screen radii, wellbore storage and skin
- Delayed drawdown response is accounted for in observation wells
- Well fields are easy to design and changes readily incorporated
- Results are accurate and optimization is fast.

# Appendix B

**Dimensionless system parameters**

Analytical well flow solutions are often presented in a dimensionless form. In this way the number of relevant parameters (hydraulic properties of the aquifer system) is reduced, the mathematics are simplified, some relations between parameters are clarified and results are more generally applicable (e.g. type curves). For well flow in a multi-layer system it is also possible to work with dimensionless drawdown, dimensionless time and dimensionless parameters, but the reduction of parameters is small and the benefit of a transformation to a dimensionless solution likewise. However, some insight into the dimensionless multi-layer parameter system may be helpful.

In a multi-layer system comprising n aquifers and (n + 1) aquitards dimensionless drawdown $s_D$ can be expressed as a function of (4n + 1) dimensionless parameters:

- $t_D$ is the dimensionless time for the pumped $k^{th}$ aquifer ($T_k t/S_k r^2$),
- 2n dimensionless leakage parameters sqrt($r^2/Tc_i$) and sqrt($r^2/Tc_{i+1}$) and
- 2n dimensionless storage parameters $S_i/S_k$ ($i \neq k$) and $S_i'/S_k$,

where $s_D$ is defined by $s_D = 4\pi T_k s/Q_k$.

An obvious conclusion from the above definition of dimensionless system parameters is that drawdown is always proportional to the pumping rate. A more interesting outcome is that these parameters show that when all transmissivities, all storativities and all discharges are multiplied by a certain factor, and all resistances are divided by the same factor, all computed drawdowns remain unaltered. We will use this feature in the discussion of the double-porosity example.

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